2006 National Finals

Level 3

Lightning Round

Question 1

Compute: $\left(5 - \sqrt{5^2 - 3^2}\right)^2$.

Question 2

The perimeter of a right, isosceles triangle is $2 + \sqrt{2}$

Find its area. Express your answer as a common fraction.

Question 3

What is the sum of the degree measures of all the interior angles of an octagon?

Question 4

A rhombus has diagonals of lengths 12 cms and 16 cms. What is the perimeter of the rhombus, in cm?

Question 5

Line y = 10 meets the y-axis at a point A and meets the line x =y at point B. If the origin is O, find the perimeter of triangle AOB. Express your answer in the simplest radical form.

Question 6

Evaluate: (a-b)/ab + (b-c)/bc + (c-a)/ca

Question 7

Each child in a family has at least 2 brothers and 2 sisters. What is the smallest number of children the family might have?

Question 8

If $x/y + y/x = 2$; what is the value of y when $x = \sqrt{2}$

Question 9

Factorize: ax + ay – 4x – 4y

Question 10

There is a two digit number that is 9 times the sum of its digits. What is this number?

Question 11

What is the smallest value of x for which $|x| \leq 6$

Question 12

The hypotenuse of a right isosceles triangle is 6. What is the length of one of the legs? Express your answer in the simplest radical form.

Question 13

The surface area of a sphere 324π square centimeters. What is volume in cubic centimeters, of the sphere? Express your answer in terms of π

Question 14

Let x be a positive number and y be its reciprocal. Compute

$$\frac{1}{x+1} + \frac{1}{y+1}.$$

Question 15

What is the ratio of $0.1\overline{6}$ to $0.8\overline{3}$? Express your answer as a common fraction.

Question 16

Positive integers B and C satisfy $B(B-C) = 17$. What is the value of C?

Question 17

A bag contains 3 blue, 4 red and 3 yellow marbles. How many blue marbles must be added to the bag for it to contain 75% blue marbles?

Question 18

Given that $\quad 4a + 5b + 7c = 13$

and $\quad 4a + 3b + c = 19,$

what is the value of $\quad a + b + c \quad$?

Question 19

Jimmi needs to be home by 6 o'clock. If he travels at an average rate of 15mph, he'll arrive one hour early, but if he travels at an average rate of 10mph, he'll arrive one hour late, How many miles away is his home?

Question 20

In the arithmetic sequence 17, *a, b, c,* 41, what is the value of *b*?