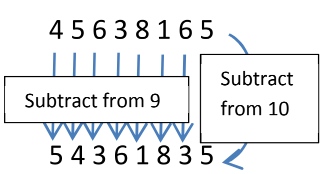
NSF Math Column – Volume 4

Speed Math Techniques

10's complement:

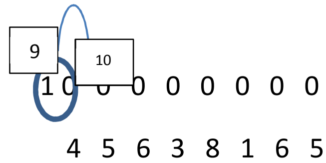
In the last few columns, we looked at a few techniques from Vedic Mathematics that would help perform multiplications faster. In this column, we will look at a very simple yet a powerful concept that has many applications. This is called the "10's complement" and it simply indicates how far behind a number is from the nearest power of 10 greater than the number itself. For example, 10's complement of 92 is 8 (nearest power of 10 greater than 92 is 100). Now, this is pretty simple for smaller numbers or numbers close to powers of 10. How do we calculate the 10's complement for 45,638,165?

Of course you can subtract the number from 100,000,000 (next power of 10) and say the answer is 54,361,835. But there is a simpler way using one of the sutras in Vedic Mathematics. The sutra says "All From 9 And The Last From 10". What this means is – subtract each digit from left to right from 9 except the rightmost digit (ones place). Subtract the ones place from 10. You now have the 10's complement. Let's use this method to verify our answer for the number above.



Even though this is simple, on careful observation this is no different than our previous method of subtracting the two numbers.

This is illustrated in the following line.



In order to perform the subtraction we borrow from the 10. Hence 10 becomes 9 and the next digit changes to 10 as shown above. We repeat this process until the ones place becomes 10. At the end of this process we are left with the following:

9 9 9 9 9 9 9 10

4 5 6 3 8 1 6 5

Now the problem is simplified to subtracting all digits from 9 and ones digit from 10!

How does this work when ones digit of the number for which you need to find the 10's complement is 0? In this case, subtracting from 10 will give you 10 and not a single digit. It is pretty straight forward if you try our traditional subtraction by borrowing. You will stop borrowing at the tens digit since both numbers have 0 in the ones place. Hence, all you have to do is ignore the zero, find the 10's complement of remaining number and add the zero to the 10's complement to get the final answer. For example, let's find the 10's complement of 45,638,160. Ignore the zero and find 10's complement for 4563816 which is 5436183. Then add the zero to the end to get the final 10's complement of 54,361,830. This can be applied to any number ending with one or more zeroes.

Practice Problems:

1. 721
2. 12344
3. 8476589
4. 18700054
5. 14983475
6. 9898989

NSF Math Bee Corner

King Midas spent (100/x)% of his gold deposit yesterday. He is set to earn gold today. What percentage of the amount of gold King Midas currently has would he need to earn today to end up with as much gold as he started?

Anytime you convert a fraction to percentage, multiply the fraction by 100. When converting a percentage to fraction, divide by 100. In this case, it is easy to first convert the percentage into a fraction.

$(100/x) / 100 = (1/x)$

Now, remaining gold is $1 – (1/x) = (x – 1)/x$.

In order to make this a whole, he needs to earn $1 – [(x – 1)/x] = 1/x$. In other words, whatever he spent!

Now, we need to find out what percentage of existing value this new value is. Important formula to remember is:

Part = (Percentage/100) * Whole

In other words,

Percentage = (Part/Whole)*100

$= \{(1/x)/[(x – 1)/x]\}*100$

$= \mathbf{[100/(x – 1)]\%}$

Martha and Mary had 375 jelly beans in all. After Mary ate 24 jelly beans and Martha ate 1/7 of her jelly beans, they each had the same number of jelly beans left. How many jelly beans did each girl have at first?

This can be solved in couple of ways (using equations or simply through fractions).

Solution 1: Let's say X and Y represent the number of jelly beans Martha and Mary had at first.

$X + Y = 375$ (Eq. 1)

Based on the next statement, we have the following equation.

$X – 24 = (6/7)Y$ (Eq. 2)

In other words, $X = (6/7)Y + 24$. Substituting this in our first equation, we get:

$(6/7)Y + 24 + Y = 375$

$(13/7)Y = 375 – 24 = 351$

$Y = 351*(7/13) =$ **189**

Therefore $X = 375 – Y =$ **186**

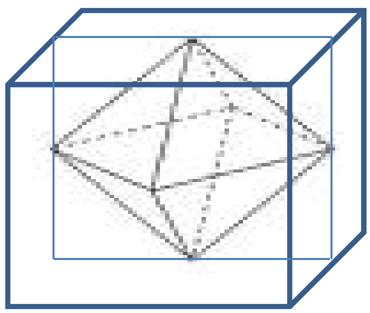
Solution 2: Let's assume the 24 jelly beans Mary ate did not exist and therefore she didn't eat any. Now the total number of jelly beans would be 351. After eating (1/7) of jelly beans, Martha will be left with 6/7 and since both have the same number at this point, Mary also has 6/7. Therefore total is (6/7) + (/6/7) + (1/7) = (13/7)

Note that we have expressed everything as a fraction of what Martha has. Now, we know that this is 351. If this total is (13/7), then Martha has 351*(7/13) = **189**

Mary has (6/7) of what Martha has plus the 24 jelly beans she ate = (6/7)*189 + 24 = **186**.

Find the volume of a regular octahedron whose vertices are at the centers of the faces of a cube.

Let's say x is the length of one of the sides of the cube. Also, we are given that this is a regular octahedron and the vertices are at the center of the cube faces. Note that the octahedron is nothing but two pyramids joined together at the base (shown below).



We can find the length of one of the bases of the pyramid using the Pythagoras theorem as follows:

$L^2 = [(x/2)^2 + (x/2)^2] = x^2/2$

$L = x/\sqrt{2}$

Now, since base of the pyramid is a square, area is L^2. Height of the pyramid is x/2.

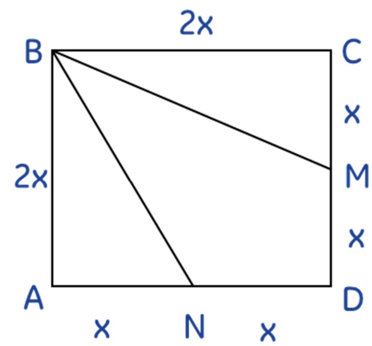
Hence volume of the pyramid = $(1/3)(L^2)(x/2)$

$= x^3/12$

Volume of the octahedron is twice the volume of the pyramid = $\mathbf{x^3/6}$

A square ABCD has line segments drawn from vertex B to the midpoints N and M of sides AD and DC respectively. Find the ratio of the perimeter of quadrilateral BMDN to the perimeter of square ABCD.

ABCD is a square and N and M are midpoints of line segments AD and DC, respectively. Let AN = x. Then ND = DM = MC = x and AB = BC = CD = DA = 2x (as shown below).



ΔBAN is a right triangle with legs BA = 2x and AN = x. Applying the Pythagoras theorem, we can find the hypotenuse BN as follows:

$$BN^2 = BA^2 + AN^2 = (2x)^2 + (x)^2 = 5x^2$$

$BN = x\sqrt{5}$.

Applying the same method for right angled triangle ΔBCM, we get $MB = x\sqrt{5}$.

Perimeter of the quadrilateral is the sum of its sides = (BN + ND + DM + MB)

$= x\sqrt{5} + x + x + x\sqrt{5}$

$= 2x(\sqrt{5} + 1)$

Perimeter of the square = 4(2x) = 8x

Hence the ratio = $2x(\sqrt{5} + 1)/8x$

$= \mathbf{(\sqrt{5} + 1) / 4}$

John and Bill toss a biased coin that has a 60% chance of coming up heads and a 40% chance of coming up tails. They flip the coin until either two heads or two tails in a row are

observed. Bill is a winner if two heads in a row are observed first. What is the probability that Bill will win?

Let P_{BH} be the probability of Bill winning after a heads is tossed and P_{BT} be the probability of Bill winning after a tails is tossed. Let's say the first toss resulted in a heads. Next toss can either be heads (3/5) or tails (2/5). In case of heads, P_{BH} = (3/5). In case of tails, $P_{BH} = (2/5)*P_{BT}$. Since these are mutually exclusive events:

$P_{BH} = (3/5) + (2/5)* P_{BT}$

Similarly, let's say the first toss is a tails. If the next toss is a tails, Bill will lose (0). If the next toss is a heads, then the chance of Bill winning is $P_{BT} = (3/5)* P_{BH}$

Since both are mutually exclusive events, we have:

$P_{BT} = (0) + (3/5)* P_{BH}$

Using the two equations, we get:

$P_{BH} = (3/5) + (2/5)*[(3/5)* P_{BH}]$

$= (3/5) + (6/25)*P_{BH}$

$(1 – (6/25))*P_{BH} = (3/5)$

P_{BH} **=** $(3/5)/(19/25)$

$= (3*25)/(5*19)$

$= 15/19$

Therefore, $P_{BT} = (3/5)*(15/19) = 9/19$.

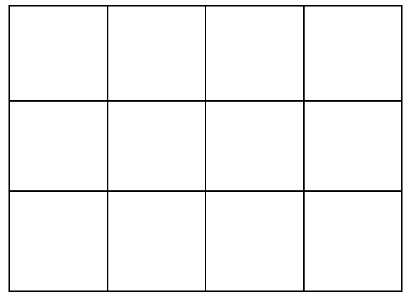
Hence, the probability that Bill wins is given as $P_B = (2/5)P_{BT} + (3/5)P_{BH}$

$= (2/5)(9/19) + (3/5)(15/19)$

$= (18 + 45)/(5*19) =$ **63/95**

Problem of the month

How many pairs of unit squares can be chosen on a 3 by 4 array of unit squares if sharing a common side is not permitted?



Would you like submit your answer? Please click on the following link: https://spreadsheets.google.com/viewform?formkey=dHR6ek5BazVnRVM3d01nbG1fNVdybXc6MA

Names of everybody who submitted correct answers will be published in the next edition!

Interested to know the solution for last column's problems? Refer to the end of this document!

Special thanks to the following Math Column contributors:

- ***Anamika Veeramani (Cleveland OH)***

For any questions or comments, please contact the team at NSFMathColumn@gmail.com

NSF Math Column – Volume 4

Answer to "Can you repeat the same with 100 as main base?" (Vol 1-3)

Step 1: Use 20 as the working base and 100 as the main base.

26	+06
19	-01

Step2: Multiply the difference.

26	+06
19	-01

	-06

Step 3: Cross add the number and difference

26	+06
19	-01

25	-06

Step 4: Calculate adjuster

Adjuster = Main Base/Working Base
= 100/20 = 5

Step 5: Divide the sum by adjuster.

26	+06
19	-01

25/(5)	-06
5	-06

Step 6:

26	+06
19	-01

5	-06
5 - 1	-06 + 100

We get 4 and 94 resulting in 494 as the answer!

Answers to Practice Problems (Vol 1-3)

1. 980
2. 2058
3. 2145
4. 7242
5. 37370
6. 44550

Answer to Problem of the month (Vol 1-3)

22 mines

<u>Solution:</u> Start with number "4". All empty squares around that number are all mines. Count the mines around number "3" and so on.

Who submitted correct answers?

- Anna Nixon (Portland, OR)
- Akhila Mamandur (Houston, TX)
- Shrutika Kumareshan (Sharon, MA)
- Anup Hiremath (Old Bridge, NJ)
- Shreya Bellur (Peoria, IL)
- Harshika Avula (San Antonio, TX)
- Vishal Gullapalli (Wayne, NJ)
- Shreyaa Raghavan (Sharon, MA)
- Maya Shankar (Bridgewater, NJ)
- Smaraki Dash (Frederick, MD)
- Varun Singh (Tampa, FL)
- Himanvi Kopuri (Denver, CO)
- Indumathi Prakash (Sharon, MA)
- Samiksha Mulpuri (Austin, TX)
- Vamsi Subraveti (Nashville, TN)
- Sanjana Rao (Suwanee, GA)
- Sruthi Parthasarathi (Mason, OH)
- Siddarth Guha (Missouri City, TX)
- Shivani Guha (Missouri City, TX)
- Ananya Yammanuru (St. Charles, IL)
- Adhith Palla (Hoffman Estates, IL)
- Keerti Priya Vajrala (Aurora, CO)
- Anusha Vajrala (Aurora, CO)
- Praniti (Jacksonville, FL)
- Dhivya Senthil Murugan (Denver, CO)
- Bhavana Muppavarapu (Buffalo Grove, IL)
- Leela Pakanati (Dunlalp, IL)
- Keerthana Chakka (Katy, TX)
- Advitheey Chelikani (Palatine, IL)
- Akash, Karanam (Sugar Land, TX)
- Sindhuja, Karanam (Sugar Land, TX)
- Shritha Gunturu (Aurora, CO)

- Sameer Lal (Macungie, PA)
- <Not Provided> (CA)
- Tanishq Kancharla (Middlebury,CT)
- Ankit Patel (Princeton, NJ)
- Kevin john (Valrico,FL)
- Anurag (Columbus,OH)
- ADITYA SRIDHAR (ISELIN, NJ)
- Deepankar Gupta (Naperville, IL)
- Sanjana Vadlamudi (Cary,NC)
- Akshay Prabhushankar (Olathe, KS)
- Navya Prabhushankar (Olathe, KS)
- Sai_Javangula (Irving,TX)

Thanks to all the participants who attempted to solve the puzzle!