NSF Math Column – Volume 8

Set Theory

Continuing with the Set Theory concepts, we'll focus on different operations that can be performed on sets. There are two main operations – ***Union*** and ***Intersection***.

Union of two or more sets is simply another set that combines all elements from the sets involved in the union operation (denoted by the symbol ∪). For example, if A = {2, 4, 6} and B = {1, 3, 5} then their union is written as follows.

A∪B = {1, 2, 3, 4, 5, 6}

As you can see, the new set combines all the elements from both the individual sets. Now, what is the union of A = {2, 4, 6, 8} and B = {2, 3, 4, 5, 6}?

A∪B = {2, 3, 4, 5, 6, 8}

Note that, we have included the common elements only once in the resulting set.

Intersection (denoted by symbol ∩) of two or more sets is simply another set that only includes the common elements. In our last example, A∩B is written as follows.

A∩B = {2, 4, 6}

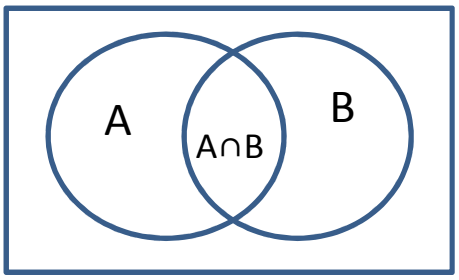
2, 4, and 6 are the only elements that appear in both sets A and B. Now, what happens if there are no common elements? This is certainly possible and in such a scenario, the resulting set is called a ***null*** set (denoted by {} or ϕ). If E is defined as a set of all positive even numbers and O as a set of all positive odd numbers, then E∩O = ϕ.

Another important concept is the ***complement*** of a set. It is simply all elements that do not belong to this set but part of the larger set. A simple example is the set of odd integers and even integers. They are complement to each other as one doesn't belong to another.

In other words, intersection of a set and its complement is always a null set!

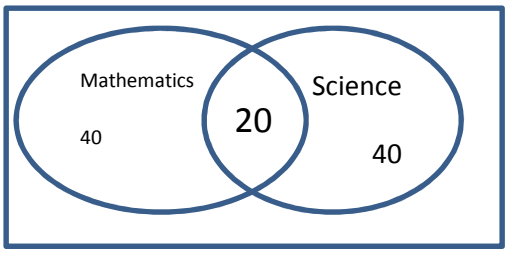
Finally, we can also depict sets pictorially and is often useful to think of them that way. This is done using ***Venn*** Diagrams.

In this, a set is typically drawn as a circle. Following is diagram representing two sets A and B.



Above diagram is a simple illustration of the sets and their operations. Circle A and Circle B individually represent the two sets. The area common to the two circles represent their intersection (shown above). All of circle A, and circle B with the common area included only once represent A∪B.

Let's look at an example. Consider a class of 100 students. Suppose 60 of them like Mathematics and some others like Science. If there are 20 students who like both Science and Mathematics, then how many students like Science only?



The representation of the sets as a Venn diagram is shown above. We know that 20 students like both subjects. So if we fill that in the middle, then we infer that 40 students like only Mathematics because it is give 60 students like Mathematics. Since we know the total number of students as 100, we can figure out the number of students who like Science only to be 40. This is a simple problem but illustrates the use of Venn diagram clearly. This can get more complicated with more than one sets.

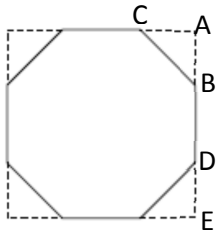
Practice Problems:

1. Set A contains 15 marbles, B contains 12 marbles and their intersection contains 8 marbles. How many marbles belong to the union of sets A and B?
2. How will you represent A∩ (B ∪ C) using a Venn diagram?

Competitive Math

(☆ *indicates difficulty level*)

A square with a side length of 2 has the corners cut off so that a regular octagon remains. Determine the perimeter of the regular octagon. ☆☆



Let x be the length of corner that is cut. AB = AC = x. Since ABC form a right-angled triangle, we can find the length BC using Pythagoras theorem.

$BC = \sqrt{(AB^2 + AC^2)} = \sqrt{(x^2 + x^2)} = \sqrt{2}x.$

Since it is a regular octagon, $BC = BD = \sqrt{2}x$.
We are given that the side of the square is 2.
Hence $x + \sqrt{2}x + x = 2$
$x = 2/(2 + \sqrt{2}) = \sqrt{2}/(\sqrt{2} + 1)$

Now, we can calculate the perimeter of the octagon as $8*BD = 8*(\sqrt{2}x)$
$= 8*\sqrt{2}*(\sqrt{2}/(\sqrt{2} + 1))$
$= 16/(\sqrt{2} + 1)$

If the age of a person 16 years ago was 5 times the current age of his son, and two years ago the sum of his age and his son's age was 30, what is the age of his son now? ☆

Let S be the age of the son and D be the age of dad. Let's try to convert information given into simple equations.

Given that 16 years ago, dad's age was 5 times his son's current age, we get the following equation.
D – 16 = 5S

Next, it is given that two years ago, sum of their ages was 30.
(D – 2) + (S – 2) = 30

Now, it's just a matter of solving the two equations.

5S + 16 = 34 – S
6S = 18
S = 3.

Thus, his son's current age is 3 and his age is 31.

NSF Math Column – Volume 8

A used car dealer sold two cars and received $1120 for each car. One of these transactions amounted to a 40% profit for the dealer, whereas the other amounted to a 20% loss. What is the dealer's net profit on the two transactions?

Important formula to remember for this problem is:

Part = (Percentage/100) * Whole

For the first car, let's say X is the actual cost. Given that the sale price is $1120 and is 40% more than the cost, we can write the following equation.

1120 = (140/100) * X
X = $800

Similarly, if Y is the actual cost of the second car then we have:

1120 = (80/100) * Y
Y = $1400

So, the total cost of the two cars = $800 + $1400 = $2200

Both were sold at the same price of $1120. Hence the total sales = 2 * $1120 = $2240

Another important formula to remember is:

Profit = Sales Revenue – Original Cost

In our case, the profit from the sale of both cars = $2240 - $2200 = $40.

A set of consecutive odd numbers starting with 1 has a sum of 400. What is the last number in the set?

Let's look at couple of different approaches to solve this problem.

Approach #1:

Let N be the highest number in the set and let's represent the set 1, 3, 5, … up to N.

We know that the sum of N consecutive numbers is N(N + 1)/2. In addition, the set of even numbers up to N can be written as 2, 4, 6, … N-1.

Sum of the even numbers = [(N – 1)(N + 1)/4]. This is derived using the same concept of sum of fist N consecutive numbers. We can write the sum of even numbers in our set as:

= 2 + 4 + 6 + … + (N – 1)
= 2[1 + 2 + 3 + … + (N – 1)/2]

In other words, it is the twice the sum of first (N – 1)/2 consecutive numbers.

We know the sum of first N numbers and sum of the even numbers up to N. Therefore, we can easily calculate the sum of the odd numbers up to N as follows. Sum of odd numbers to N = (Sum of N consecutive numbers) – (Sum of even numbers to N)

= [N(N + 1)/2] – [(N – 1)(N + 1)/4]
= [(N + 1)/2][N – (N – 1)/2]
= $[(N + 1)/2]^2$

We are given that this is 400. Hence, [(N + 1)/2] = 20 and N = **39**.

Approach #2:

This can also be solved very quickly using the fact that square of any number P can be expressed as a sum of first P odd numbers.

Therefore $P^2 = 400$ and $P = 20$

This indicates that we have 20 odd numbers in our set and hence, last number would be **39**.

Tyler needs to score 100 points for his final Mathematics test of the year to improve his average score from 76 to 79. How many mathematics tests are there in a year?

Important formula to remember for this is calculating average. Average of N numbers is given by the following formula.

Average of N numbers = (Sum of N numbers)/N

In other words, sum of N numbers is the product of average and N.

Approach #1:

Let's say there are N tests total in our case. Given that the average for (N – 1) tests is 76, total score from the (N – 1) tests will be 76(N – 1). Score for the N^{th} test is given as 100 and final average is given as 79.
Now, the average of all N test scores is:

[76(N – 1) + 100]/N = 79
76N – 76 + 100 = 79N
24 = 3N
N = 8

Approach #2:

This can also be solved without writing any equations. Note that the increase in average is nothing but the amount of score above the current average from the last test equally divided among all the tests. Since the last test score if 24 points above average, and the average increased by 3 we need to have 8 tests in total.

Problem of the month

Samir is a short put champion and is planning to participate in the next Olympic Games. He starts to practice and completes 49 throws from a center point in different directions. When he started to measure all the distances, he found that every throw landed in a spot 1-ft away from the previous one. In addition, the line connecting center point and the new spot always was perpendicular to the line connecting the new spot and the previous spot. If his last throw distance was 1 feet, how much distance was his very first throw?

Would you like submit your answer? Please click on the following link:
https://spreadsheets.google.com/viewform?formkey=dHR6ek5BazVnRVM3d01nbG1fNVdybXc6MA

Names of everybody who submitted correct answers will be published in the next edition!

Interested to know the solution for last column's problems? Refer to the end of this document!

For any questions or comments, please contact the team at NSFMathColumn@gmail.com

NSF Math Column – Volume 8

Answers to Practice Problems (Vol 1-7)

1. Set of whole numbers W = {0, 1, 2, … }
2. M = {5, 10, 15, … , 95} and |M| = 19.
3. Real numbers between 1 and 10.

Answer to Problem of the month (Vol 1-7)

0

Solution:

Let's say a, b, and c are the first three numbers in the data sequence. Given the following:

(a + b + c) = 2(a + c)

In other words, b = a + c. Since this is true for any set of 3 adjacent numbers we can write the sequence as follows.

a, a+c, c, -a, -a-c, -c, a, a+c, c, …

Note: Each number after the first three are derived from the previous two such that the given condition is satisfied.

We can notice the pattern emerging. The pattern repeats itself every 6 numbers in the sequence. We are given that 115^{th} value is 16. Based on the sequence, 115^{th} will be "a" and hence we have a = 16. If 115^{th} value is a, then 114^{th} value in the sequence is "-c". Given that a + (-c) = 0, we have a = c. Therefore, c = 16. With the values for a and c, we can now identify any number in the sequence. Finally, we need to find the sum of 250^{th} and 253^{rd} values. 250 when divided by 6 leaves a reminder of 4. Hence it's value is same as "-a" or "-16". 253^{rd} value will be "a" or "16". Hence their sum is 0.

Who submitted correct answers?

- Rekha ,chandak (Dallas, TX)
- Anita,Patel (Princeton, NJ)
- Aditya,Sridhar (Iselin, NJ)
- Nikhil,Parchuri (Princeton, NJ)
- Krishna,Bharathala (Fremont,CA)
- Madhavi,Reddy (Jacksonville, FL)
- Tarang,Saluja (Nashua, NH)
- Jay,Gurrala (San Antonio, TX)
- Shruthi,Santhanam (Suwanee, GA)
- Simoni,Maniar (Grapevine,TX)
- anika,ramachandran (Cupertino, CA)
- yash,chandak (Dallas, TX)
- Mana,Singri (Southlake, TX)
- Gayathri,Srirajan (Waukegan, IL)
- Siddarth,Guha (Missouri City, TX)
- Shivani,Guha (Missouri City, TX)
- Jarnail,Singh (Cleveland, OH)
- Shaila,Patankar (Marlboro, NJ)
- Ilakiya ,Udhayakumar (Metuchen, NJ)
- sanmeshkumar ,Udhayakumar (Metuchen, NJ)
- Harshika,Avula (San Antonio, TX)
- Nina,Vendhan (Denver, CO)
- Akshaj,Kadaveru (Fairfax, VA)
- Chetana,Ramesh (Sycamore, OH)
- Sruthi,Parthasarathi (Mason, OH)
- Anup,Hiremath (Old Bridge, NJ)
- Desigamoorthy,Nainar (Champaign, IL)
- Akshay,Venkat (Leawood, KS)
- Rachana,Madhukara (San Diego, CA)
- Tanushree,Pal (Ventura, CA)
- Samhitha,Somavarapu (Manassas, VA)
- sreekar ,chitti (Bangalore, Karnataka, India)
- Tanishq,Kancharla (Middlebury, CT)
- Shalini,Dangi,MV
- Akshay,Prabhushankar (Olathe, KS)
- Hemanth,Chitti (Bangalore, India)
- rohan,rereddy (Garland, TX)
- Nishant,Chittari (Columbus, OH)
- Sudharani,Tangirala (Campbell, CA)
- Ravikiran,Komirisetty (Irvine, CA)
- preetha,saravanan (Denver, CO)
- Sneha,Reddy (Jacksonville, FL)

- Bhavya,Bansal (Plano, TX)
- Suraj,Shroff,Nashua. NH
- Bhavana,Muppavarapu (Buffalo Grove, IL)
- Indumathi,Prakash (Sharon, MA)
- Ankit,Patel (Princeton, NJ)
- Anna,Nixon (Portland, OR)
- sayuj,shajith (Suwanee, GA)
- Saahith,Mummadi (Novi, MI)
- Deepankar,Gupta (Naperville, IL)
- Nikhilganeesh,Varadarajan (Cary, NC)
- Ashwath,Raj (San Diego, CA)
- Kannan,nagarajan (Weston, FL)
- Preetham,Bachina (Pleasanton, CA)
- Priya,Parchuri (Princeton, NJ)
- Nymisha,Mattapalli (Herndon, VA)
- Himanvi,Kopuri (Denver, CO)
- Gayatri,Ramesh (Cincinnati, OH)
- Madhu,Gogulapati (Bakersfield, CA)
- Shalini,Agarwal (Overland Park, KS)
- Vibha,Agarwal (Overland Park, KS)
- Rao,Allu (St. Louis, MO)
- Keerthana,Chakka (Katy, TX)
- vidhya,kannan,FL
- Janani,Sridhar (Cupertino, CA)
- Suganth,Kannan (Weston, FL)
- Arunandhi,Rathinam (Rochester Hills, MI)
- Mrugank,Gandhi (Aurora, IL)
- Ravi,Ponnusamy (Franklin, WI)
- Shraman,Sen (Morrisville, NC)
- Geetanjali,Khanna (Piscataway, NJ)
- Prasanna,Reddy (Somerset, NJ)
- Sunita,Jagana (Alpharetta, GA)
- Kruthi and Bhargav,Annigeri (Portland, OR)
- Ashutosh,Reddy (Oxford, AL)
- Saisuki,Putumbaka (Basking Ridge,NJ)
- Meghana,Annambhotla (Southbury, CT)
- Rashmi,Madhukara (San Diego, CA)
- Anish,Neervannan (Irvine, CA)
- Ramanan,Srirajan (Waukegan, IL)
- Sreenu,Pamidi (Shrewsbury, MA)
- Roshan ,George (Charlottesville, VA)
- Satvik,Reddy (Jacksonville, FL)
- Dhivya ,Senthil Murugan (Denver, CO)
- Gargi,Sadalgekar (East Windsor, NJ)
- Vishal,Purohitham (Sanford, FL)
- Shrey,Agarwal (Valrico, FL)
- Esha,Shakthy (Suwanee, GA)
- Indira,Kadoor (Lilburn, GA)
- Shaheel,Mitra (Cincinnati, OH)
- Teja,Veeramacheneni (Fremont, CA)
- Adithya Mummidi,Mummidi (San Antonio,TX)
- Harshayu,Girase (Union City, CA)
- Anjali,Gupta (Edison, NJ)
- Shashank,Mahesh (Tampa, FL)
- Manish,Suryapalam (Downingtown, PA)
- Arvind ,Chandaka (Hillsborough, NJ)
- Asha,Chandaka,Hillsborough NJ
- Nihar,Vallem (Denver, CO)
- Thushar,Mahesh (Tampa, FL)
- Vishik,Bhalla (Nashua, NH)
- Shreyaa,Raghavan (Sharon, MA)
- Shruthi,Satyanarayana (Fremont, CA)
- Anish,Bose (Austin, TX)
- Shritha,Gunturu (Aurora, CO)
- Arnav,Singh (Tampa, FL)
- Aayush,Singh (Tampa, FL)
- Shraeya,Madhu (Clarksburg, MD)
- Vamsi,Subraveti (Nashville, TN)
- Sraavya,Pinjala (Salt Lake City, UT)
- Mahesh,Ganti (Austin, TX)
- Sayak,Chatterjee (Winchester, MA)
- Laasya,Renganathan (Cary, NC)
- Praneeth,Prathi (shrewsbury,MA)
- Satvik,Kolluri (Austin, TX)
- Keerti ,Vajrala (Aurora, CO)
- Anusha,Vajrala (Aurora, CO)
- Varun,Singh (Tampa, FL)
- Soumika,Guduru (San Diego, CA)
- Neha,Dudipala (Columbus, OH)
- Sanjna,Khanna (Piscataway,NJ)
- Anish,Ravipati (Dublin, OH)
- Neha,Seshadri (Bangalore, India)
- Arvind,Chava (Herndon, VA)
- Naveen,SKV (Devon,PA)
- Sahana,Aiyer (Herndon, VA)
- shiva,senthilkumar (knoxville, TN)
- Ashish,Kumbhardare (West Chester , PA)
- Aarush,Prasad (Tampa, FL)
- Samiksha,Mulpuri (Austin, TX)
- Sonali,Razan (Shrewasbury, MA)
- Arnav,Jagasia (Villanova, PA)

- Aditya Vargheese (Overland Park, KS)
- Mohana,Aravamudham,US
- Sneha,Prasath (Westford, MA)
- Nitin,Pinnamaneni (San Ramon, CA)
- Rahul,Madala (Chantilly, VA)
- THARINI,RAMAKRISHNAN (Portland, OR)
- Pranav,Kotilingam (North Brunswick, NJ)
- Vidhya,S (Voorhees, NJ)
- Bharat,Kabra (Ediosn, NJ)
- Santhi,Senthilkumar (Bloomington, IL)
- Dhruv,Puri (Austin, TX)
- Bharati,Dalal,NC
- Prithiv,Prithiv (Bloomington, IL)
- Maya,Shankar (Bridgewater, NJ)
- Pranav Rekapalli (Atlanta, GA)
- Akshar Thakkar (Naperville, IL)
- Sruti Ganesh (Novi, MI)
- Anjali Nambrath (Malboro, NJ)
- Pratiik Kaushik (McMurray,PA)
- Mala Kaushik (McMurray,PA)
- Mahima Parupalli (Tampa, FL)
- Anoop Naravaram
- Shreya Bellur (Dunlap, IL)
- Ananya Yammanuru (St Charles, IL)
- Sahana Kumar (San Diego, CA)
- Roshan Toopal (Chandler, AZ)
- Nikita Mullick (Brookfield, WI)
- Sneha Koneru (San Antonio, TX)
- Nina Koneru (San Antonio, TX)
- Rachana Chilakapati (Randolph, NJ)
- Arya Koneru (San Antonio, TX)
- Rahul Dev (Bangalore, India)
- Suresh Koneru (San Antonio, TX)
- Rishita Saladi (Campbell, CA)

Thanks to all who attempted to solve the problem of the month. The Math Column team is looking forward to your continued interest and increased participation.