NSF Math Column – Volume 12

Competitive Math

(☆ *indicates difficulty level*)

After Ed eats 20% of a pie and Ann eats 40% of a pie, Ed has twice as much pie left as Ann. Find Ed's original amount of pie as a percentage of Ann's original amount? ☆

Let's say Ed's pie was X units and Ann's pie was Y units. After Ed eats 20% of the pie, he is left with 80% or 0.8X. Similarly, if Ann eats 40% of her pie, then she is left with 60% or 0.6Y. Given that the ration of Ed to Ann after they eat is 2:1, we can write the equation below.

$0.8X = 2(0.6Y)$
$X = 1.5Y$

In other words, Originally Ed had **150%** of what Ann had.

A fair coin is flipped six times. What is the probability to have three heads up and three heads down ? ☆

Each coin flip ends up as either H or T. Thus there are $2^6 = 64$ total possible outcomes from the 6 tosses. Now, we need to figure out the number of favorable outcomes, which is three H and three T. We can find this out in couple of different ways. Enumerating the possibilities, we have the following:
{HHHTTT, HHTHTT, HHTTHT, HHTTTH, HTHHTT, HTHTTH, HTHTHT, HTTTHH, HTTHHT, HTTTHH, TTTHHH, TTHTHH, TTHHTH, TTHHHT, THHHTT, THHTTH, THHTHT, THTTHH, THTHHT, THTTHH}
Total number of favorable outcomes = 20.

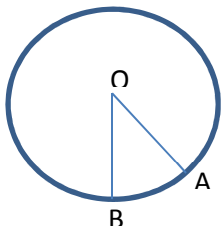
Another approach is to treat this same as number of different arrangements of 6 letters (3 H and 3 T). So, there are 6! possible arrangements. But the arrangements of the three H's are indifferent and we should not over count. Similar argument applies for T's as well. Hence we divide the total possible arrangements by 3! for each of H and T as follows:

$6!/[3!3!] = 6!/(3! * 3!) = 5!/3! = 20$

Therefore, the probability is nothing but the ratio of favorable outcomes to the total outcomes.

P(3 H, 3 T) = 20/64 = **5/16**.

Two points A and B lie on the circumference of a circle. If the radius is 9 ft and 4 in, and length of arc AB is length 5 ft and 10 in what is the measure of angle AOB (in degrees) .? ☆☆



Note that in any circle with radius R, the circumference is $(2\pi)R$. In other words, if we draw an arc from the centre of the circle that covers the entire circle (360^o), then the length of the arc is $(360^o)*R$ or $(2\pi)R$.

We have to be careful with radians and degrees. 2π radians = 360^o or $1^o = (\pi/180)$ radians.

Refer to the diagram shown above. We have O as the center of the circle and AB is the arc.

In our case, we are given the length of arc as 5 ft 10 in = 70 in. Radius is given as 9 ft 4 in = 112 in.

Therefore, measure of angle in radians = 70/112. In degrees, the angle will be (70/112)(180/π) = **225/2π degrees**.

The numbers 1,2,…,n are written down in random order. What is the probability that n-1 and n are written next to each other? (Give your answer in term of n.)

Let's take an example and see if we can derive a pattern. If we take the numbers 1,2 and 3 then we have the following possible arrangements.
123, 132, 213, 231, 312, 321.
There are a total of 6 or 3! arrangements. Now, out of these we are interested only in the following four arrangements (2 and 3 must be next to each other).
123, 132, 231, 321

We notice that "23" appears 2 times or (n-1) times. Similarly, "32" appears 2 times or (n-1) times.

By extending this to n elements, (n-1) and n can be arranged in 2(n-1) ways starting from position 1.
Total arracngements of n elements is n!. Thus the probability of having (n-1) and n next to each other is given by the ratio of the number of favorable outcomes to the number of total possible outcomes = 2(n-1)/n! = **2/n**.

Tim's car gets 3 more miles per gallon during highway driving than it does during city driving. On a recent trip, Tim drove 136 miles on the highway and 155 miles in the city, using a total of 9 gallons of gasoline. How many miles per gallon does Tim's car get during city driving?

Let H be the miles per gallon on highway and C be the miles per gallon in the city. Given H = C + 3.
Tim drove 136 miles on the highway. This would have consumed 136/H gallons of gas. Tim also drove 155 miles in the city resulting in usage of 155/C gallons of gas. Since Tim used a total of 9 gallons of gas, we have the following equation.

(136/H) + (155/C) = 9
136C + 155H = 9HC

136C + 155(C + 3) = 9(C + 3)C
136C + 155C + 465 = $9C^2$ + 27C
$9C^2$ – 264C – 465 = 0

$3C^2$ – 88C – 155 = 0
(3C + 5)(C – 31) = 0

Since C cannot be negative, C has to be **31 miles per gallon**.

Niki just completed a 10 mile bike trip. If she had been able to ride 5 miles per hour faster, she would have completed her trip 1 hour earlier. Find her speed in miles per hour?

Important formula to remember in dealing with speed, time, distance is d = rt, where d is the distance, r is the speed rate, and t is the time taken.

We know that the distance is 10 miles. Let's say r is the rate at which Niki completed the bike trip. If t is the time taken to complete, we have 10 = rt.

Next, it is given that if Niki increased her speed by 5 miles per hour, then she would have completed 1 hour earlier.
Based on this information, we can write the following equation.

$10 = (r + 5)*(t – 1)$

From the two equations, we have $rt = (r + 5)*(t – 1)$
Also, substituting for t from the first equation, we get the following quadratic equation.
$r^2 + 5r – 50 = 0$
$(r + 10)(r – 5) = 0$

Thus, her rate of speed is 5 miles per hour.

Problem of the month

Let x and y be drawn (with replacement) from {1,2,3,...,99} such that each ordered pair (x, y) is equally likely. Given that x + y is even, determine the probability that the sum of the units digits of x and y is less than 10.

Would you like submit your answer? Please click on the following link:
https://spreadsheets.google.com/viewform?formkey=dHR6ek5BazVnRVM3d01nbG1fNVdybXc6MA

Names of everybody who submitted correct answers will be published in the next edition!

Interested to know the solution for last column's problems? Refer to the end of this document!

For any questions or comments, please contact the team at NSFMathColumn@gmail.com

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Answer to Problem of the month (Vol 1-11)

621 cm (569 cm if the ball didn't travel between 12th and the 1st person)

Solution:

Using trigonometric rations, you can identify the side of the polygon hence the circumference of the polygon.

Who submitted correct answers?

- Akshaj Kadaveru (Fairfax VA)
- Kannan nagarajan (Weston FL)
- Abhishek Allamsetty (Herndon VA)
- Ajit Kadaveru (Fairfax VA)
- preetha saravanan (Denver CO)
- Himanvi Kopuri (Denver CO)
- vijaya Madala (Chantilly VA)
- Monal Garg (East Brunswick NJ)
- Anju Garg (East Brunswick NJ)
- Siddarth Guha (Missouri City TX)
- Viknesh Baskar (Rochester NY)
- Sai Allu (St. louis MO)
- Krishna Mahankali (Bridgewater NJ)
- Rajasekhar Kothuri CA
- Anusha Allamstty (Herndon VA)
- Anna Nixon (Portland OR)
- shiva senthilkumar (Knoxville TN)
- Sameer Lal (Macungie PA)
- Neha Khandelwal (Haymarket VA)
- vidhya kannan (Riverview FL)
- Dhivya Senthil Murugan (Denver CO)
- Nishant Chittari (New Albany OH)
- BHARATI DALAL (Ahmedabad India)
- Sampreeti Chowdhuri (San Diego CA)
- preetham bachina (Pleasanton CA)
- Shiny Antony Pembroke Pines
- Deepankar Gupta (Naperville IL)
- Anika Ramachandran (Cupertino CA)
- Shalini Dangi Mission Viejo
- Aditya Sriram (Corvallis OR)
- Pranav Nagarajan (Fremont CA)
- Anup Hiremath (Fremont CA)
- Anita Virjala (Santa Clara CA)
- Sushovan Guha (Missouri City TX)
- Raju Sunkasari (Cary NC)
- subhash siripurapu (San Antonio TX)
- Maya Shankar (Bridgewater NJ)
- Savan Kumar (Lawrence MA)
- Ramesh Madesiah (Johns Creek GA)
- Akshara Kannan (Riverview FL)
- Anish Madala (Chantilly VA)
- Simoni Maniar (Grapevine TX)
- Divya Gubba TX
- Keshav Mallidi (Roanoke VA)
- Rudrakshi Dasika (Tucson AZ)
- Shashank Mahesh (Tampa FL)
- Sundar Sankaran (Voorhees NJ)
- Tarang Saluja (Nashua NH)
- Anita Patel (Princeton NJ)
- Hemanth Chitti (Bangalore India)
- Sudharani Tangirala (Campbell CA)
- Aryaman Khandelwal (Allentown PA)
- Meena Shankar (Bridgewater NJ)
- rekha chandak (Dallas TX)
- Keshav Kotteswaran DE
- Gayathri Srirajan (Waukegan IL)
- Rupesh Chinta (Dayton NJ)
- Rajesh Nair (Gurnee IL)
- Rahul Madala (Chantilly VA)
- Vinita Cheepurupalli (Columbia SC)
- Vishal Purohitham (Sanford FL)
- Tanushree Pal (Ventura CA)
- Adi Bulusu (Moorestown NJ)
- Sowmya Bulusu (Moorestown NJ)
- Yash Somaiya (Fairfax VA)
- Pranay Malempati (Newark DE)
- Shwetark Patel (Herndon VA)
- Devendra Patel (Herndon VA)
- Krishna Bharathala (Fremont CA)
- Akaash Sanyal (Cary NC)
- Sunita Upadhyayula (Plainfield IL)
- Anirudh Kuchibhatla (Hyderabad India)
- Meghana Gudavalli New Jersey
- Hemal Dalal NC
- Janani Sridhar (Cupertino CA)

- Jayanth Gunda (Hyderabad India)
- Meghana Annambhotla (Southbury CT)
- Nalini Boda (Lewis Center OH)
- Shraeya Madhu (Clarksburg MD)
- Soumika Guduru (San Diego CA)
- Alap Sahoo (Bakersfield CA)
- Veenu Gupta (Tampa FL)
- Sudha Shenoy (Bear DE)
- komal handoo (Tampa FL)
- Anitha Ramakodi (Parsippany NJ)
- sreekar chitti (Bangalore India)
- praneeth prathi (Shrewsbury MA)
- Rohan Balel (Chicago IL)
- Pranav Upadhyayula (Plainfield IL)
- Arvind Chava (Herndon VA)
- Nihar Vallem (Aurora CO)
- Siya Kalra (Johns Creek GA)
- Himani Kalra (Johns Creek GA)
- Shreya Bellur (Dunlap IL)
- Ashok Bhatia (San Diego CA)
- Indumathi Prakash (Sharon MA)
- Pranav Kotilingam (North Brunswick NJ)
- Mythri Challa (Coralville IA)
- pratiik kaushik McMurray
- Sreenu Pamidi (Shrewsbury MA)
- Nitin Sadalgekar (East Windsor NJ)
- Jayashree Vedamurthy (Dayton NJ)
- Sankar Mahadevan (Dayton NJ)
- Pavani Samala (West Chester PA)
- vishruti ganesh (San Ramon CA)
- Subashni Rajiv (Los Angeles CA)
- Sushil Upadhyayula (Plainfield IL)
- nirjara pillai (Greenville SC)
- Nymisha Mattapalli (Herndon VA)
- Kala Nathan (Chicago IL)
- Vanitha Sankaranarayanan (Walnut Creek CA)
- Neha Seshadri (Novi MI)
- Saisuki Putumbaka (Basking Ridge NJ)
- Shreeman Nimmagadda (Ann Arbor MI)
- Surya Jaladi (St.Louis MO)
- Risheet Nair (Marlton NJ)
- Sanjana Eranki (Nesconset NY)
- Neela Ramesh (Cincinnati OH)
- Chetana Ramesh (Cincinnati OH)
- Anirudh Chowlur (Tamarac FL)
- Abhishek Routray (Katy TX)
- Aayush Gupta (Saratoga FL)
- Samruddhi Hande (San diego CA)
- Anjali Nambrath (Marlboro NJ)
- yadunandan pillai (greenville SC)
- Keerti Vajrala (Aurora CO)
- Nithin Gudavalli (Franklin park NJ)
- Bindu Seshadri USA
- Vishal Gullapalli (Wayne NJ)
- A Thakkar (Naperville IL)
- Shaheel Mitra (Cincinnati OH)
- Varun Singh (Tampa FL)
- shruthi prabaharasundar (Troy OH)
- Geetanjali Khanna (Piscataway NJ)
- YASH CHANDAK (Irving TX)
- Gargi Sadalgekar (East Windsor NJ)
- Aditya Kumar (Herndon VA)
- Shreyaa Raghavan (Sharon MA)
- Anusha Vajrala (Aurora CO)
- Meghana Nambiyur (Round Rock TX)
- Manisha Solipuram (Novi MI)
- Laasya Renganathan (Cary NC)
- Mukta Valluru (Austin TX)
- Sahana Aiyer (Fairfax VA)
- Shritha Gunturu (Aurora CO)
- Nisha Goel (Winchester MA)
- Pratham Goswami (Rochester MI)
- Nitya Guduru (Bensalem PA)

Thanks to all who attempted to solve the problem of the month. The Math Column team is looking forward to your continued interest and increased participation.