NSF Math Column

Speed Math Techniques

Multiplication using addition and subtraction - II:

In the last issue, we looked at numbers that are close to as well as less than base 10 or 100. This article will extend the same approach to numbers above and below the base. Let's look at an example:

What is 9 x 12?

Since both numbers are close to 10, we'll use that as our base. Next, we need to calculate the difference between each number and the base number and write it to the side as shown below.

9 -1
12 +2

Next, multiply the difference. In this case it is (-1) x (+2) = -2. Now write the product below the numbers.

9 -1
12 +2

-2

Finally, add the numbers across and write the sum below as shown below.

9 -1
12 +2
----
11 -2

So far the steps were the same as we have seen before. Now last step is to convert the negative number into a positive number by borrowing from the previous (left) digit.

9 -1
12 +2
----
11 -2
11 - 1 -2 + 10

Note that we are borrowing from the tens place. Hence we subtract 1 from the 10's place and add 10 to the ones place. Once we borrow, we have the following:

9 -1
12 +2
----
10 8

Product is 108. We have the answer!

Let's try another example. What is 98 x 103?

Step 1: Select the base. Since the numbers are close to 100, we choose the base as 100.

Step 2:

98 -02
103 +03
----
101 -06

Step 3:

98 -02
103 +03
----
101 -06
101 - 1 -06 + 100

Note that we are borrowing from the hundreds place in this case.

98 -02
103 +03
----
100 94

Hence the answer for 98 x 103 is 10094.

Practice Problems:

(Hint: Choose 10 as the base for 1 through 4 and 100 for 5 through 8)

1. 7 x 12
2. 15 x 9
3. 12 x 12
4. 10 x 14
5. 102 x 99
6. 90 x 109
7. 85 x 102
8. 111 x 103

Competitive Math

A magician places n cards numbered from 1 to n face down on a table where n is chosen by a person in the audience. He calls up n people from the audience. He instructs the first person to blindfold him and then turn over every card, the second person to turn over every second card, the third person to turn over every third card and so on. After the nth person, he asks several people to call out the number of a card (between 1 and n inclusive) and he tells them correctly whether the card is face up or face down. How does he know?

When dealing with variable numbers like this, it is always a good to understand the behavior for small numbers and then try to extrapolate it. Let's pick the value of n as 10. Following table shows the sequence of actions on various cards.

	Card Numbers									
People	1	2	3	4	5	6	7	8	9	10
1	**U**	U	U	U	U	U	U	U	U	U
2		D		D		D		D		D
3			D			U			D	
4				**U**				U		
5					D					U
6						D				
7							D			
8								D		
9									**U**	
10										D

Now, we can see the pattern. All cards started face down. But which ones are "face up"? Card numbers 1, 4, and 9. What are these? If you said "Perfect squares" you are absolutely right! You can try the same for a different value of n and will get the same pattern. So for any value of n and cards marked with consecutive numbers from 1 through n, all perfect squares from 1 to n will be face up.

If there were 500 cards and 500 people, then how many people turned over card 420?

If a 7-letter word is constructed at random from the English alphabets, what is the probability that no letter occurs more than once? Assume that all sequences of 7-letters are equally likely.

As we all know, the probability is calculated by first identifying the total number of favorable outcomes and then dividing that by the total number of possible outcomes.

For the total number of possible outcomes, we calculate the number of 7-letter words possible (including words where letters repeat). In other words, we calculate the number of possibilities assuming the letters are replaced after every selection.

Total # of events = 26^7

Now, to look at the total number of favorable outcomes we need to find the possibilities where the same letter doesn't repeat. In other words we have to identify the possibilities assuming no replacement.

# of favorable events = 26x25x24x23x22x21x20

Hence the probability that a 7-letter word is constructed at random where no letter repeats is given by:

P = (26x25x24x23x22x21x20) / 26^7
= 0.4128 (rounded)

In other words, there is a 41.28% chance that a 7-letter word formed at random has no letters repeating more than once.

Quick Teasers

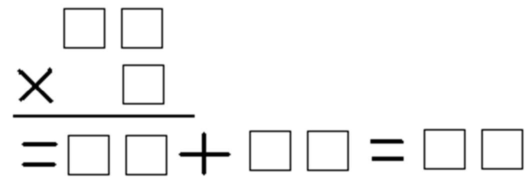
Can you find a way to express the number 100 using six 9s?

APPLE PICKERS: If five apple pickers can pick five apples in five seconds, how many apple pickers would it take to pick sixty apples a minute?

PAGE NUMBERS: You pull out a page from a newspaper and find that pages 8 and 21 are on the same sheet. From that, can you tell how many pages the newspaper had?

Problem of the month

Missing Numbers: Nine empty boxes must be filled with digits 1 to 9. Can you work out the way to place the numbers so that the mathematical operations are correct?



Would you like submit your answer? Please click on the following link:
https://spreadsheets.google.com/viewform?formkey=dHR6ek5BazVnRVM3d01nbG1fNVdybXc6MA

Names of everybody who submitted correct answers will be published in the next edition!

For any questions or comments, please contact the team at NSFMathColumn@gmail.com

NSF Math Column

Answer to "Can you prove why this works?" (Vol 1-1)

For any two single digit numbers a, b we will find out a x b.

Step 1: Select the base. Since these are single digit numbers we can choose 10 as the base.

Step 2: Find the difference between the numbers and the base number and write next to the corresponding numbers.

a (a – 10)
b (b – 10)

Step 3: Multiply the difference and write below the numbers as shown.

a (a – 10)
b (b – 10)

(a-10)(b-10)

Step 4: Cross add the numbers and write the sum below the numbers.

a (a – 10)
b (b – 10)
----
b + (a – 10) (a-10)(b-10)

Here [b + (a – 10)] is the tens place of the final answer and [(a – 10)(b – 10)] is the units place.

Hence our answer is:
10[b + (a – 10)] + [(a – 10)(b – 10)]
$= \cancel{10b} + \cancel{10a} - \cancel{100} + ab - \cancel{10a} - \cancel{10b} + \cancel{100}$

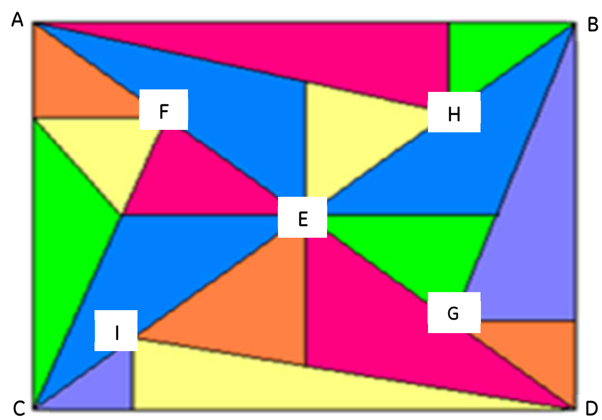
As you can see we end up with the product of a and b!

Answers to Practice Problems (Vol 1-1)

1. 56
2. 81
3. 54
4. 90
5. 9801
6. 8730
7. 7735
8. 7744

Answer to Problem of the month (Vol 1-1)

38 triangles



Area CED, DEB, and BEA each has 7 triangles. Area AFC has 9 triangles. In addition, we have 4 triangles using the four vertices:

BCD, DAB, BCA, DAC

Finally, we have the 4 triangles DCF, CAH, ABG, and BDI giving us a total of 7 + 7 + 7 + 9 + 4 + 4 = 38.

Who submitted correct answers?

- Dhivya Senthil Murugan (Denver, CO)
- Sriraj Atluri (Weston, FL)
- Karan Menon (Edison, NJ)
- Bhargav

Thanks to the remaining 53 participants who attempted to solve the puzzle!