NSF Math Column

Speed Math Techniques

Multiplication using addition and subtraction:

This article explains a simple method to calculate product of numbers. This is based on an approach described in Vedic Mathematics. Important step in this method is to identify the correct base. Let's look at an example:

What is 9 x 8?

For those who know the multiplication tables, it is easy to say 72. Now let's try the new method to calculate the product. Since both numbers are close to 10, we'll use that as our base. Next, we need to calculate the difference between each number and the base number and write it to the side as shown below.

```
9  -1
8  -2
```

Next, multiply the difference. In this case it is -1 x -2 = 2. Now write the product below the numbers.

```
  9  -1
  8  -2
-------------
     +2
```

Finally, add the numbers across and write the sum below as shown below.

```
  9  -1
    ↗
  8  -2
-------------
  7   2
```

We have the answer!

Let's try another example. What is 98 x 96?

Step 1: Select the base. Since the numbers are close to 100, we choose the base as 100.

Step 2: Find the difference between the numbers and the base number and write next to the corresponding numbers. Note that the difference is written as 2-digit numbers. This is because our base is 100.

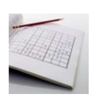
```
    98        -02
    96        -04
-----------------------
              +08
```

Step 3: Cross add the numbers and write the sum below the numbers.

```
    98       ↗-02
    96 ↗      -04
-----------------------
    94        +08
```

Hence the answer for 98 x 96 is 9408.

Can you prove why this works?



Practice Problems:

(Hint: Choose 10 as the base for single digit products and 100 for two-digit products)

1. 7 x 8
2. 9 x 9
3. 6 x 9
4. 10 x 9
5. 99 x 99
6. 90 x 97
7. 85 x 91
8. 88 x 88

Competitive Math

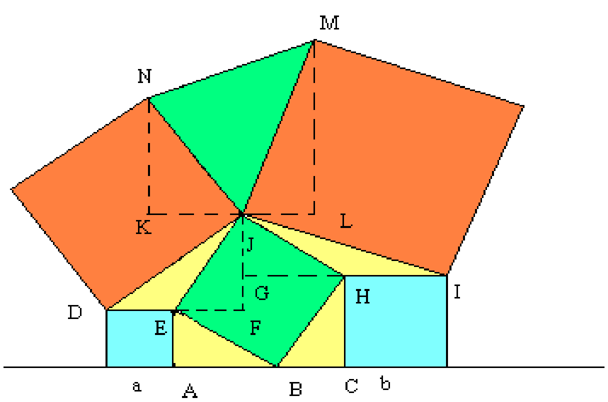
Near the end of a party, everyone shakes hands with everybody else. A straggler arrives and shakes hands with only those people whom the straggler knows. Altogether sixty-eight handshakes occurred. How many other people at the party did the straggler know?

***Answer:* 2**

Solution*: If all N people at a party shake hands with all others present then N (N- 1)/2 handshakes will take place altogether. Hence, the number of handshakes before the straggler's arrival must have been sixty-six because that is the largest plausible value less than sixty-eight. The straggler must have known two other people at the party. Constructing a table of possible values of N(N-1)/2 clarifies that sixty-six is the only plausible number.*

n	*7*	*8*	*9*	*10*	*11*	*12*	*13*
N(N-1)/2	*21*	*28*	*36*	*45*	*55*	*66*	*78*

Show that the area of the green square is equal to the area of the green triangle.



Solution:

Let the small blue square have a side 'a' and big blue square 'b'.

∆ABE ≅ ∆CHB, AE = CB = a and AB = CH = b
∆ABE ≅ ∆FJE, AE = EF = a and AB = FJ = b
Then DF = DE + EF = a + a = 2a
∆DFJ ≅ ∆NKJ, NK = DF = 2a and JK = JF = b

Proceeding similarly,

∆BCH ≅ ∆JGH, JG = BC = a and GH = CH = b
Then GI = GH + HI = b + b = 2b
∆JGI ≅ ∆JLM, JL = JG = a and LM = GI = 2b
Area of trapezoid KLMN = (1/2) x KL x (KN + LM)
= (1/2) x (b + a) (2a + 2b)
where KL = KJ + JL = b + a
= (1/2) (a + b) x 2(a + b)
= $(a + b)^2$
Area of ∆JKN = (1/2) x JK x KN
= (1/2) x b x 2a = ab
Area of ∆JLM = (1/2) x JL x LM
= (1/2) x a x 2b = ab
Area of ∆JMN = area of KLMN – area of ∆JKN – area of ∆JLM

= $(a + b)^2 - ab - ab$
= $a^2 + 2ab + b^2 - ab - ab$
= $a^2 + b^2$
= $AE^2 + AB^2$ (from ∆ABE)
= BE^2

= area of square BEJH

This proves that the area of the green square is equal to the area of the green triangle.

The supplement of an angle is 78 degrees less than twice the supplement of the complement of the angle. Find the measure of the angle.

Answer: *26°*

Solution:

Let A = measure in degrees of the angle.
180 – A = be the supplement of this angle.
90 - A = be the complement of the angle.

Then,
$(180 - A) + 78 = 2\,(180 - (90 - A))$
$258 - A = 2\,(90 + A)$
$258 - A = 180 + 2A$
$78 = 3A$

A = 26 degrees

Interesting Number Paradox

Have you thought of classifying numbers as interesting or dull?

When you think of interesting numbers, the ones that usually come to mind are π, ϕ, e…

But 2 is interesting as it is the only even prime, 3 is interesting as it is the smallest odd prime, 6 is interesting as it is the smallest perfect number etc.

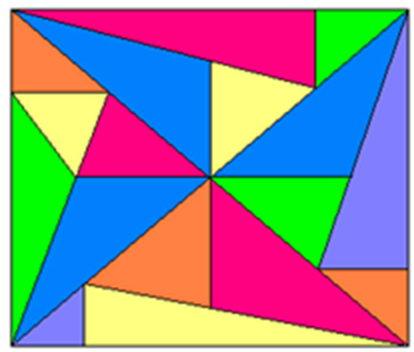
Is it possible that all natural numbers are interesting?

Let us try by assuming the opposite, that not all numbers are interesting. But then in this set of uninteresting or dull numbers there would be a smallest number, say N, which would make it special or interesting as the smallest dull number N. This is a contradiction to our assumption that there are non-interesting numbers.

Hence we arrive at an interesting paradox - all numbers are interesting!!

Problem of the month

Triangle Town: How many triangles can you count in this square?



Would you like submit your answer? Please click on the following link:
https://spreadsheets.google.com/viewform?formkey=dHR6ek5BazVnRVM3d01nbG1fNVdybXc6MA

Names of everybody who submitted correct answers will be published in the next edition!

For any questions or comments, please contact the team at NSFMathColumn@gmail.com