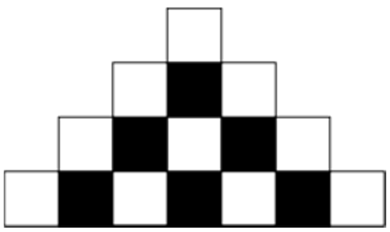
NSF Math Column

Competitive Math

(*indicates difficulty level*)

A "stair-step" figure is made of alternating black and white squares in each row. Rows 1 through 4 are shown. All rows begin and end with a white square. What is the number of black squares in the 37th row is? (AMC8)



This is an example of correctly identifying the pattern. Let's take a look at the number of squares in each row. Each row has two additional squares compared to previous row except for the first row. We can write the pattern as follows:

$N = 1 + 2(N – 1)$

In the 37^{th} row, we have $1 + 2(37 – 1) = 73$ squares. Now, to find the number of black squares we can find from the figure that the number of black squares is one more than the previous row starting from 0 black squares in row 1. Therefore, 37^{th} row should have 36 black squares. Differently, we can see that the number of white squares is always one more than the black square. Hence number of black squares will be $(N – 1)/2$ where N is the number of squares in the row. Since we have 73 squares in the 37^{th} row, the number of black squares will be $(73 – 1)/2 = 72/2 =$ **36**.

X, Y, and Z can cut a lawn in 2 hours, 3 hours, and 2 hours respectively. If X cuts the lawn for ½ hour, and then Y cuts the same lawn for 1 hour, how much time will Z take to cut the remaining lawn ?

Let's find the rate at which each of them work. Rate at which X cuts the lawn is ½ per hour. Similarly, for Y it is 1/3 per hour and for Z it is ½ per hour.

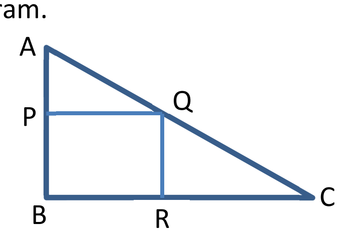
Now, X starts cutting the lawn for ½ hour and cuts ¼ of the lawn. Y then cuts for 1 hour and does 1/3 of the lawn.

Thus X and Y cuts a total of ¼ + 1/3 = 7/12 of the lawn.

Now, 5/12 of the lawn is left over. Time taken by Z to finish cutting the remaining lawn will be (5/12)/(1/2) = **5/6** hours or **50** minutes.

Through a point on the hypotenuse of a right triangle, lines are drawn parallel to the legs of the triangle so that the triangle is divided into a square and two smaller right triangles. The area of one of the two small right triangles is m times the area of the square. Find the ratio of the area of the other small right triangle to the area of the square. (AMC 10)

Let's visualize the problem using the following diagram.



Area of the square is PQ^2. Area of the triangle APQ is (1/2)*PQ*AP. It is given that this area is m times the area of the square.

(1/2)*PQ*AP = m*(PQ^2)
PQ = AP/(2m)

Area of the other triangle QRC = (1/2)*RC*QR = (1/2)*RC*[AP/(2m)], since QR = PQ.

Also, note that the angles AQP and QCR are the same thus making the two triangles similar. Therefore, (AP/QR) = (PQ/RC).

RC = (PQ*QR)/AP

Substituting this in the area of the triangle, we have:
(1/2)*[(PQ*QR)/AP]*[AP/(2m)]
= (1/2)(PQ*QR)/(2m)

Since PQ*QR is the area of the square, the ratio of the area of triangle to area of square is **1/(4m)**.

Problem of the month

Mady has an infinite number of balls and empty boxes available to her. The empty boxes, each capable of holding four balls, are arranged in a row from left to right. At the first step, she places a ball in the first box of the row. At each subsequent step, she places a ball in the first box of the row that still has room for a ball and empties any previous boxes. How many balls in total are in the boxes as a result of Mady's 2010th step?

Would you like submit your answer? Please click on the following link:
https://spreadsheets.google.com/viewform?formkey=dHR6ek5BazVnRVM3d01nbG1fNVdybXc6MA

Names of everybody who submitted correct answers will be published in the next edition!

Interested to know the solution for last column's problems? Refer to the end of this document!

Special thanks to the following Math Column contributors:

- ***Poorva Gupta (POM Contributor)***

For any questions or comments, please contact the team at NSFMathColumn@gmail.com

Answer to Problem of the month (Vol 1-12)

2401/4901 or 48.99%

Solution (***Source***: ***AoPS***)**:**

In order for the sum (x + y) to be even, we know that either both x and y are even or both x and y are odd. In the set of numbers from 1 through 99, we have 50 odd numbers {1,3,5,…,99} and 49 even numbers {2,4,6,…,98}.

In order to find the probability, we have to know the total possible outcomes. In our case, a pair (x, y) such that both x and y are odd can be selected in 49x49 = 2401 ways. Similarly, a pair (x, y) such that both x and y are even can be selected in 50x50 = 2500 ways. Therefore, total possible outcomes = 2401 + 2500 = 4901.

Next, we need to find the number of favorable outcomes. In our case, favorable outcomes are the pairs where the units digits of x and y add up to a number less than 10.

In case where both x and y are odd, then possible units digits are 1, 3, 5, 7, 9. So, possible sums that result in a value less than 10 are {(1,1), (1,3), (1,5), (1,7), (3,1), (3,3), (3,5), (5,1), (5,3), (7,1)}. It is also clear that for each of the units digits identified above there are 10 different numbers to choose. For example, {1, 11, 21, 31, 41, 51, 61, 71, 81, 91} all have units digit of 1. Also, we have 10x10 = 100 possible pairs of (x, y) for the units digit of 1. Hence, in total there are 10x10x10 = 1000 possible pairs where both numbers in the pair are odd.
Let's look at the case where both x and y are even. Possible values for units digits are 0, 2, 4, 6, 8 and possible pairs such that sum is less than 10 are {(0,0), (0,2), (0,4), (0,6), (0,8), (2,0), (2,2), (2,4), (2,6), (4,0), (4,2), (4,4), (6,0), (6,2), (8,0)}. For the pair (0,0) there are 9x9 = 81 possibilities. Every pair where one of the number end with a 0 we have 9 choices for number ending in 0 and 10 choices for number ending on other even digits. Since there are 8 such pairs, we have a total of 8x9x10 = 720 possibilities. For all other pairs, we 10 choices for both numbers and since there are 6 such pairs, we have 6x10x10 = 600 possibilities. Hence total number of possibilities for pairs of even numbers is 81 + 720 + 600 = 1401. Therefore, number of favorable outcomes = 1000 + 1401 = 2401.

Probability = # of favorable outcomes/# of total outcomes = 2401/4901 or 48.99%.

Who submitted correct answers?

- Shalini Dangi (Mission Viejo)
- Akshaj Kadaveru (Fairfax VA)
- Tanushree Pal (Ventura CA)
- Jay Gurrala (San Antonio TX)
- Ajit Kadaveru (Fairfax VA)
- Jayanth Gunda (Hyderabad India)
- Aaditya Singh (Mclean VA)
- Sankar Mahadevan (Dayton NJ)
- Tarang Saluja (Nashua NH)
- Meghana Gudavalli (NJ)
- Siddarth Guha (Missouri City TX)
- Sushovan Guha (Missouri City TX)
- Anjali Nambrath (Marlboro NJ)
- Anika Ramachandran (Cupertino CA)
- Yash Chandak (Dallas TX)
- rekha chandak (Dallas TX)
- Neha Khandelwal (Haymarket VA)
- Thushar Mahesh (Tampa FL)
- Shashank Mahesh (Tampa FL)
- vijaya madala (Chantilly VA)
- Alap Sahoo (Bakersfield CA)
- Kannan Nagarajan (Weston FL)
- Rahul Madala (Chantilly VA)
- Anish Madala (Chantilly VA)

- Shraeya Madhu (Clarksburg MD)
- Rakesh Gupta (Saratoga CA)
- Dhivya Senthil Murugan (Denver CO)
- Anupam Sharma (Haymarket VA)
- Geetanjali Khanna (Piscataway NJ)
- Shruthi Santhanam (Suwanee GA)
- Mythri Challa (Coralville IA)
- Anirudh Kuchibhatla (Hyderabad India)
- Pavani Samala (West Chester PA)
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- Himanvi Kopuri (Denver CO)
- Indumathi Prakash (Sharon MA)
- Nithin Gudavalli (USA)
- Savan Kumar (Westford)
- Hemanth Chitti (Bangalore India)
- Sreekar Chitti (Bangalore India)
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- Sushil Upadhyayula (Plainfield IL)
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- Ritika Revoori (Sharon MA)
- Shwetha Mudalegundi (Cumming GA)
- Anna Nixon (Portland OR)
- Neha Seshadri (Novi MI)
- Maya Shankar (Bridgewater NJ)
- Surya jaladi (St.Louis MO)
- Soumika Guduru (San Diego CA)
- Akash Karanam (Sugar Land TX)
- Akilan Murugesan (Carmichael CA)
- Abhishek Allamsetty (Herndon VA)
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- A Thakkar (Naperville IL)
- SUGANTH KANNAN (Weston)
- Pranav Senthilkumar (CA)

Thanks to all who attempted to solve the problem of the month. The Math Column team is looking forward to your continued interest and increased participation.