NSF Math Column – Volume 3

Speed Math Techniques

Multiplication using addition and subtraction - III:

In the last issue, we looked at multiplying numbers that are above and below the base numbers 10 or 100. This article will extend the same approach but with bases that are multiples of the "main" bases (powers of 10) referred to as "working" bases. Let's look at an example:

What is 26 x 19?

Since both numbers are close to 20, we'll use that as our "working" base. It is also important to choose the "main" base at this point. We'll choose 10 as our main base. Next, we need to calculate the difference between each number and the working base number and write it to the side as shown below.

```
26     +6
19     -1
```

Next, multiply the difference. In this case it is (+6) x (-1) = -6. Now write the product below the numbers.

```
26     +6
19     -1
-----------------
       -6
```

Note that if we had selected 100 as our main base, we would write "06" instead of "6" above. Finally, add the numbers across and write the sum below as shown below.

```
26     +6
19     -1
------------------
25     -6
```

So far the steps were the same as we have seen before. Now we identify the ratio of the base numbers referred to as the "adjuster" as follows:

Adjuster = Main Base/Working Base

= 10/20 = 1/2

Now we divide the sum above by the adjuster as shown below.

```
26              +6
19              -1
---------------------------
25/(1/2)        -6
50              -6
```

Now the last step is to convert the negative number into a positive number by borrowing from the previous (left) digit.

```
26        +6
19        -1
--------------------
50        -6
50 - 1    -6 + 10
```

Note that we are borrowing from the tens place. Hence we subtract 1 from the 10's place and add 10 to the ones place. Once we borrow, we have the following:

```
26        +6
19        -1
--------------------
49        4
```

Product is 494. We have the answer!

This can be applied to any number as the "working" base that is a multiple of a "main" base (powers of 10).

Can you repeat the same with 100 as main base?

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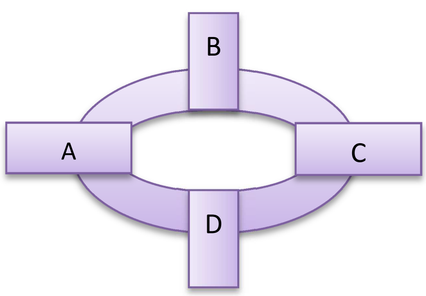
Practice Problems:

1. 28 x 35
2. 42 x 49
3. 39 x 55
4. 71 x 102
5. 185 x 202
6. 198 x 225

Competitive Math

John enters a circular road that has four exits A, B, C and D as shown below. He enters through A. When reaching each exit, he is equally likely to either exit or continue (including turning around). What is the probability that John will exit at A?



Let's say P(XY) = Probability of exiting at Y while currently at X. At point A, chance of John exiting at A, moving to B, or moving to D are equally likely. Following are three possibilities:

- Exit at A now. This probability is 1/3.
- Go to B. In this case, the probability to go towards B is 1/3. But our goal is to find the probability that John exits at A. So we need to find the probability of John exiting at A from B. So, total probability if (1/3)*P(BA).
- Go to D. Similar to the previous option, this probability is (1/3)*P(DA).

These are ***mutually exclusive*** events and results in the following equation:

$$P(AA) = (1/3)*(1) + (1/3)*P(BA) + (1/3)*P(DA) \qquad [Eq.1]$$

Similarly, at exits B, C, and D we get the following equations (Note: At B,C, and D if John decides to exit then the chance of exiting at A is 0).

$$P(BA) = (1/3)*(0) + (1/3)*P(AA) + (1/3)*P(CA) \qquad [Eq.2]$$
$$P(CA) = (1/3)*(0) + (1/3)*P(BA) + (1/3)*P(DA) \qquad [Eq.3]$$
$$P(DA) = (1/3)*(0) + (1/3)*P(AA) + (1/3)*P(CA) \qquad [Eq.4]$$

We can also infer the fact that P(BA) = P(DA) since all chances are equally likely. Now rewriting equations (1) and (3), we have the following:

$$P(AA) = (1/3) + (2/3)*P(BA) \qquad [Eq.5]$$
$$P(CA) = (2/3)*P(BA) \qquad [Eq.6]$$

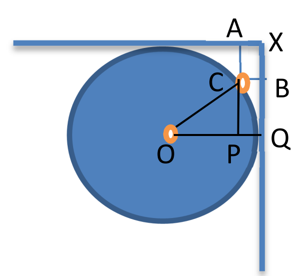
Using these in equation (4), we get:

$$P(BA) = (1/3)*[(1/3) + (2/3)*P(BA)] + (1/3)*[(2/3)*P(BA)]$$

Solving this will give us P(BA) = (1/5). Substituting in equation (5) will give us the answer.

P(AA) = (7/15) or 46.67%

A circular table is pushed into a corner of a rectangular room so that it touches both walls. A point on the edge of the table between the two points of contact is 2 inches from one wall and 9 inches from the other wall. What is the radius of the table?



Let A and B be the points where table touches the walls. Let C be the point on the circle between A and B and X represent the corner of the room. We are given the following:

BC = 2 inches
AC = 9 inches

From the figure, PQ = BC = 2 inches.

Let r be the radius of the circle.

$OP = r - 2$

Note that AP is same as the radius r. Hence we can derive the following:

$CP = r - 9$
$OC = r$

Now, using Pythagoras theorem for the right-angled triangle OCP, we can write:

$OC^2 = OP^2 + CP^2$

$r^2 = (r - 2)^2 + (r - 9)^2$
$r^2 = r^2 - 4r + 4 + r^2 - 18r + 81$
$r^2 = 2r^2 - 22r + 85$
$r^2 - 22r + 85 = 0$

Solving this equation, r = 17 or r = 5.

Since AP is same as radius and AC is 9, radius cannot be 5. Hence radius of the circle is 17 inches.

Problem of the month

The 8×10 grid below has numbers in half the squares. These numbers indicate the number of mines among the squares that share an edge with the given one. Squares containing numbers do not contain mines. Each square that does not have a number either has a single mine or nothing at all. How many mines are there?

	1		1		2		2		1
1		2		3		2		3	
	3		2		3		2		2
1		3		2		1		3	
	3		2		1		1		2
2		4		1		1		2	
	3		3		2		3		1
1		2		2		3		2	

Would you like submit your answer? Please click on the following link:
https://spreadsheets.google.com/viewform?formkey=dHR6ek5BazVnRVM3d01nbG1fNVdybXc6MA

Names of everybody who submitted correct answers will be published in the next edition!

Interested to know the solution for last column's problems? Refer to the end of this document!

For any questions or comments, please contact the team at NSFMathColumn@gmail.com

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Answer to "If there were 500 cards and 500 people, then how many people turned over card 420?" (Vol 1-2)

Basically, it is all the factors of 420. To find the number of factors, we can do the following:

- Express the number as power of prime factors.

  $2^2 x 3^1 x 5^1 x 7^1$

- Increment each exponent by 1 and multiply to get number of factors.

  (2+1)*(1+1)*(1+1)*(1+1) = 24

Number 420 has 24 factors and thus 24 people turned over card 420!

Answer to "Quick Teasers" (Vol 1-2)

 You can do this multiple ways. Following is one way to express 100 using six nines.

99 + 99/99

APPLE PICKERS: Five. The same pickers who can pick 5 apples in five seconds can pick sixty apples in sixty seconds!

PAGE NUMBERS: Since there are seven pages before page 8, there has to be seven pages after page 21.The newspaper has 28 pages.

Answers to Practice Problems (Vol 1-2)

1. 84	2. 135
3. 144	4. 140
5. 10098	6. 9810
7. 8670	8. 11433

Answer to Problem of the month (Vol 1-2)

$$\begin{array}{r} \boxed{1}\,\boxed{7} \\ \boxed{4} \\ \hline \end{array}$$

$$= \boxed{6}\,\boxed{8} + \boxed{2}\,\boxed{5} = \boxed{9}\,\boxed{3}$$

Who submitted correct answers?

- Siddarth Guha (Missouri City, TX)
- Dhivya Senthil Murugan (Denver, CO)
- Akshay (Portland, Oregon)
- Abhinav R Karthikeyan (Clarksburg, MD)
- Indumathi Prakash (Sharon,MA)
- Ananya Yammanuru (St. Charles, IL)
- Shreyaa Raghavan (MA)
- Harshika Avula (San Antonio, TX)
- Maya Shankar (Bridgewater, NJ)
- Divya Bachina (Sharon,MA)

Thanks to all the participants who attempted to solve the puzzle!