NSF Math Column – Volume 5

Speed Math Techniques

Subtraction:

In the last article we looked at a simple concept known as “10’s complement” that indicates how far behind a number is from the nearest power of 10 greater than the number itself. In this article, we’ll examine one use of it. For example, what is 156725 – 8888?

```
               1
1  5  6  7  2  5
         8  8  8  8
-----------------------
                     7
```

In the traditional method, we borrow 1 from the tens place as 5 is less than 8. Now we have 15 and subtract 8 and write 7 for the units place in the answer as shown above. Now we need to remember that the tens place has only 1. Since 1 is less than 8, we borrow from the hundreds place and we now have 11. At this point, we subtract 8 and write 3 for the hundreds place in the answer.

```
             1
1  5  6  7  1  5
         8  8  8  8
-----------------------
                  3  7
```

We’ll repeat the steps to end up with the answer 147837. With just two numbers, we have to keep track of digits borrowed and what is left in the previous digits etc. This gets only worse with large numbers or when you have more than 2 numbers to deal with.

Let’s try the same using our 10’s complement. In this case, we first find the 10’s complement of the number to be subtracted – “8888”, which happens to be 1112. Now, we add the 10’s complement to the other number.

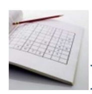
```
1  5  6  7  2  5
      1  1  1  2
------------------------
1  5  7  8  3  7
```

Subtract the power of 10 that was used to find the 10’s complement. In our case, it was 10,000.

So, the answer is 147837!

Note that the subtraction is split into a simple addition and a subtraction. Subtracting power of 10 from a number is lot easier as you just saw.

Can you prove why this works? Also, can you think how to do this for multiple numbers each having its 10’s complement from different powers of 10?



Practice Problems:

1. 115 – 95
2. 275 – 6
3. 17855 – 367
4. 98275 – 77777
5. 347658 – 345 – 88889

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NSF Math Bee Corner

If a natural number P is divisible by 15 and 28, then it must also be divisible by the natural number Q where $30 \le Q < 48$. Find the sum of all possible values of Q satisfying this condition.

The fact that P is divisible by 15 and 28 means it is also divisible by their LCM. Let's find the LCM of 15 and 28 by listing out their prime factors.

$15 = 3 * 5$
$28 = 2 * 2 * 7$

Since there are no common factors, LCM is simply the product of all their factors

LCM (15,28) = 2 * 2 * 3 * 5 * 7 = 420.

Based on this, we can say that P is a multiple of 420. If P is divisible by Q, then Q must be a factor of 420. We can solve this in couple ways.

Solution 1: Since 420 ends with a 0, the factors have to end in 0, 2, or 5. With that in mind, we have the following possible numbers – 30, 32, 35, 40, 42, 45. Now, if you see which one is a factor of 420, we get 30, 35, and 42.

Solution 2: We list out all combinations of the ***prime factors*** and pick the ones that satisfy the condition for Q.

2	3 x 5	2 x 3 x 7 ←
3	3 x 7	2 x 5 x 7
5	5 x 7 ←	2^2 x 3 x 5
7	2^2 x 3	2^2 x 3 x 7
2 x 2	2^2 x 5	2^2 x 5 x 7
2 x 3	2^2 x 7	2 x 3 x 5 x 7
2 x 5	2 x 3 x 5 ←	2^2 x 3 x 5 x 7

We have 30, 35 and 42 as factors satisfying the given condition for Q.

Hence, sum of the factors is 30 + 35 + 42 = 107.

What is the remainder when $5^{17} + 7^{19}$ is divided by 8?

Analyze the problem using modular arithmetic, noting the patterns:

5 mod 8 = 5	7 mod 8 = 7
5^2 mod 8 = 1	7^2 mod 8 = 1
5^3 mod 8 = 5	7^3 mod 8 = 7
5^4 mod 8 = 1	7^4 mod 8 = 1
.	.
.	.
.	.
5^{17} mod 8 = 5	7^{19} mod 8 = 7

So when ($5^{17} + 7^{19}$) is divided by 8, the remainder is 4.

30 men can complete a job in 12 days if each man works 8 hours a day. If the job is to be completed in 10 days with 24 men working at the same rate, how many hours must they work each day?

Note that each worker spends 12 x 8 = 96 hours. So, total amount of hours needed to complete the job is 30 x 96 = 2880.

Now, if the same job needs to be completed by 24 men, then each would have to spend 2880/24 = 120 hours. Given that we have only 10 days to complete the job, every worker must spend 12 hours each day in order to complete the same job in 10 days.

Marley had $45 and Molly had $30. Then Marley and Molly spent some money in the ratio 2:1 respectively. Now the ratio of money Marley has to the amount of money Molly has is 1:2. How much money did Marley spend?

Let x be the amount of money Molly spent. Then Marley would have spent 2x. After they spent the money, following is what is left.

Marley = 45 – 2x

Molly = 30 – x

Given that Molly has twice the amount of money after they spent, we have the following equation.

$30 - x = 2(45 - 2x)$

$30 - x = 90 - 4x$

$3x = 60$

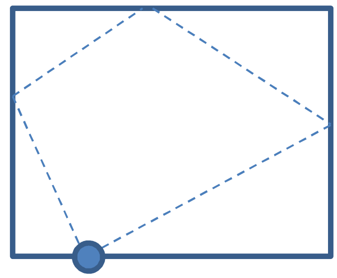
$x = 20$

In other words, Molly spent $20. Hence Marley spent $40.

Alternate approach:

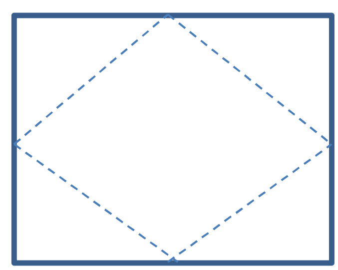
Since Marley spends $2 for every $1 spent by Molly, the difference between them changes by $1. Now, let's say Molly doesn't spend any money. In that case, for Molly to end up with twice the amount of money Marley has, Marley should spend $30. In other words, this represents all three parts in the ratio 2:1. Therefore one part is $10. So, Molly spends in multiples of $10 and Marley spends in multiple of $20. Since Marley can only spend either $20 or $40, we can easily find that she has to spend $40 to satisfy the post condition of Molly ending up with twice the amount.

John stands against one wall of a square room with walls of length 4 meters each. He kicks a frictionless, perfectly elastic ball in such a way that it bounces of the three other walls once each and returns to him (diagram not geometrically accurate). How many meters does the ball travel?



If you carefully notice, the ball travels the length and width of the square twice starting from one point and getting back to the same. Hence, the ball travels 8 meters horizontally and 8 meters vertically. So, we can calculate the total distance travelled as 8√2.

Alternate approach:



2m

2m

The path of the bouncing ball is nothing but a variation of the one shown above where it hits midpoint of every side of the square. So, we can calculate the length from one midpoint to the next one using Pythagoras theorem as 2√2. Hence the total distance travelled is 4 times this value - 8√2.

If you roll three fair dice, what is the probability that the product of the three numbers rolled is prime?

Remember that a prime number has only 1 and itself as the factors. So, any outcome where all three numbers are different will not result in the product being a prime. In addition, two out of the three has to be a 1. The combinations that would result in the product being a prime are:

{ (1,1,2), (1,1,3), (1,1,5), (1,2,1), (1,3,1), (1,5,1), (2,1,1), (3,1,1), (5,1,1)}

So, the number of favorable outcomes = 9. Total number of possible outcomes from rolling three dice is 6x6x6.

Now, we can calculate the probability as follows:

P = # of favorable outcomes/Total # of possible outcomes

$P = 9/6^3$

$= 1/24$

Problem of the month

One hundred people are standing in a line and they are required to count off in fives as 'one, two, three, four, five', and so on from first person in the line. Anyone who counts 'five' walks out of the line. Those remaining repeat this procedure until only four people remain in the line. What was the original position in the line of the last person to leave?

Would you like submit your answer? Please click on the following link:
https://spreadsheets.google.com/viewform?formkey=dHR6ek5BazVnRVM3d01nbG1fNVdybXc6MA

Names of everybody who submitted correct answers will be published in the next edition!

Interested to know the solution for last column's problems? Refer to the end of this document!

Special thanks to the following Math Column contributors:

- ***Anamika Veeramani (Cleveland OH)***
- ***Srinivasa Rao Karanam (Sugarland, TX)***

For any questions or comments, please contact the team at NSFMathColumn@gmail.com

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Answer to the practice problems (Vol 1-4)

1. 279
2. 87656
3. 1523411
4. 81299946
5. 85016525
6. 101011

Answer to Problem of the month (Vol 1-4)

We will build a model in stages. First identify each square with a number. Number the squares in the first row 1, 2, 3, and 4; those in the second row 5, 6, 7, and 8; and those in the third row 9, 10, 11, and 12. Now for each square we can count the number of squares that can be paired with it. For example, square number 1 could be paired with squares numbered 3, 4, 7, 8, 9, 10, 11, and 12. Now it is a matter of systematically counting the number of compatible squares for each square.

By systematically counting each compatible square, we arrive at the following configuration:

9	8	8	9
8	7	7	8
9	8	8	9

The numbers given in the above configurations are actually twice as many as the possible pairs, since each number represents each element in all possible pairs. For example, since the top-left and top-right squares can each be paired with nine other squares, “9’ was placed in both. However the top-left and top-right squares can be paired with each other, so that pair was counted twice. Upon inspection we see that each pair has been counted twice in our scheme.

Therefore the correct number of pairs of square is one half of 98, or 49

Who submitted correct answers?

- Shritha Gunturu(Aurora, CO)
- Keerti Vajrala(Aurora, CO)
- Anusha Vajrala(Aurora, CO)
- Siddarth Guha(Missouri City, TX)
- Shivani Guha(Missouri City, TX)
- Anna Nixon(Portland,OR)
- Shreya Bellur(Dunlap, IL)
- Akshaj Kadaveru(Fairfax, VA)
- Ananya Yammanuru(St. Charles, IL)
- Navya Prabhushankar(Olathe, KS)
- AkshayPrabhushankar(Olathe, KS)
- Vishal Gullapalli(Wayne, NJ)
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- ADITYA SRIDHAR(ISELIN, NJ)
- Sruthi Parthasarathi(Mason, OH)
- Ankit Patel(Princeton, NJ)
- Mrugank Gandhi(Aurora, IL)
- Akash Karanam (Sugar Land, TX)
- Sanjana Vadlamudi(Cary,NC)
- Kundan Chintamaneni(Middletown, MD)
- Sindhuja Karanam (Sugar Land, TX)
- Sayuj shajith(Suwanee,GA)
- Keerthana Chakka(Katy,TX)
- Tanishq Kancharla(Middlebury,CT)
- Maya Shankar(Bridgewater, NJ)
- Himanvi Kopuri(Denver,CO)
- Bhavana Muppavarapu(Buffalo Grove, IL)
- Vamsi Subraveti(Nashville,TN)
- CHAITU KONJETI(NASHVILLE,TN)
- Nisha Goel (MA)
- Aishwarya Ilangovan (Westerville)
- Amogh Gaitonde (CA)

- Praneeth prathi (Shrewsbury,MA)
- Subhiksha Balasundar(Dallas,TX)
- Varun Singh(Tampa, FL)
- Maithri Kora(Columbus,OH)
- Hrishikesh Kommu(Houston,TX)
- Desigamoorthy Nainar(champaign,IL)
- Hemanth Chitti(Bangalore,India)
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- Aarush Prasad(Tampa, FL)
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- Harshika Avula(San Antonio, TX)
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- Sreekar chitti(bangalore,India)
- Shruthi Santhanam(Suwanee,GA)
- Ilakiya Udhayakumar(Metuchen,NJ)
- Aarush Garg(Farmington Hills, MI)
- Shreyaa Raghavan(Sharon,MA)
- Roshan George(charlottesville, VA)
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- Anjali Gupta (Edison, NJ)
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Thanks to all who attempted to solve the problem of the month. The Math Column team is looking forward to your continued interest and increased participation.