NSF Math Column – Volume 11

Competitive Math

(★ *indicates difficulty level*)

In a school there are 25 students who are eligible to be part of student council. Student council can be formed either with 4 or 5 students. What percentage is the number of ways of forming a 4-person student council is compared to the number of ways of forming a 4-person student council? ★★

Important fact to remember is the number of ways you can choose "k" objects out of a pool of "n" objects where k <= n. This is also written as "n Choose k" and is given by the formula:
C (n, k) = n!/[k!(n – k)!]

In our case, we can choose 4 out of 25 in 25!/(4!21!) ways. Similarly, we can choose 5 out of 25 in 25!/(5!20!) ways.
Therefore, the ratio can be found as follows.

C(25,4)/C(25,5) = {25!/(4!21!)}/{25!/(5!20!)}
= (5!20!)/(4!21!)
= 5/21

In other words, C(25,4) is **23.8%** of C(25,5).

Joe was practicing for his marathon and measured his speed at different times. The readings showed that he averaged 10 mph on level ground, 40% of it running uphill and 120% of it running downhill. If he ran 5 miles in level ground, 2 miles uphill and 3 miles downhill what was his average speed from start to finish (rounded to the tenths)? ★★

Formula for average speed is given as follows:

Average speed = (Total distance travelled)/(Total time taken)

In our case, we know the total distance travelled = 5 + 2 + 3 = 10 miles. But we don't know the total time. We can calculate that using his average speed. Since he travelled at 10 mph in level ground for 5 miles, time taken should be 5/10 = 0.2 hours. He averaged 40% of his level ground speed in uphill = 0.4*10 = 4 mph. At this rate, he would have covered 2 miles in 2/4 = 0.5 hours. Finally, he averaged 120% of his level ground speed during downhill = 1.2*10 = 12 mph. At this rate, he would have covered 3 miles in 3/12 = 0.25 hours.
Total time taken = 0.2 + 0.5 + 0.25 = 0.95 hours

Average speed = 10/0.95 = **10.5 mph**

Sara wanted to buy fish for her aquarium. She determined that 9 goldfish plus 3 angelfish would cost $8.70. If she bought 6 angelfish and 4 goldfish, the cost would be $8.30. What is cost of one goldfish? ★

Let X be the cost of one goldfish and Y be the cost of one angelfish. Since 9 goldfish and 3 angelfish cost $8.70, we have the following equation.
9X + 3Y = 8.70

We can reduce this further to 3X + Y = 2.90. Similarly, based on the information that 6 angelfish and 4 goldfish cost $8.30, we have the following equation.
6Y + 4X = 8.30

Using the reduced equation, we can re-write the second equation as:

6(2.90 – 3X) + 4X = 8.30
17.40 – 18X + 4X = 8.30
9.10 = 14X
X = 0.65

Cost of one goldfish = $0.65.

In how many ways can you arrange the word SIXTEEN such that the two Es are not next to each other?

First let's find the total number of possible combinations of the word. There are 7 letters in the word and hence 7! Possible ways to rearrange the letters. But note that there are two E's and their arrangement is double counted (two E's are not distinguishable). Therefore, total possible arrangements is 7!/2 = 2520 ways.
Now, in order to find the number of arrangements where the two E's are not next to each other, it is easy to find the opposite (two E's are next to each other) and then subtract from the total possibilities. This is called the ***complementary counting*** technique.

Now, the two E's can be together in one of the 6 positional arrangements starting from first position and hence total arrangements is 6! = 720.

Therefore, total number of ways to arrange the letters so that the two E's don't appear next to each other = 2520 – 720 = **1800** ways.

A piece of wire 72 cm long is cut into two equal parts and each is formed into a circle. What is the sum of the areas of the two circles?

Important formulae to remember are the perimeter and area of a circle. Perimeter of a circle with radiaus r is given by $2\pi r$ and area is given by πr^2.

In tour example, the wire of length 72 cm is cut into two equal parts. Each piece is 36 cm long which is same as the perimeter of each circle.
$2\pi r = 36$
$r = 36/2\pi = 18/\pi$

Area of a circle = $\pi r^2 = \pi(18/\pi)^2 = 324/\pi$.

Therefore, the sum of the areas of two circles = $2(324/\pi) = 648/\pi$ cm^2.

If the two lines x + 3y = 6 and kx + 2y = 12 are perpendicular, what is the value of k?

Fact to remember about perpendicular lines is that the slope of one is negative reciprocal of the other.
Let's rewrite the equations as follows.
$y = (-1/3)x + 2$

Any line can be represented as y = mx +c where m is the slope of the line. In this example, slope of the above line is (-1/3).
Similarly, we can rewrite the second equation as $y = (-k/2)x + 6$.

Now, for the two lines to be perpendicular, following condition has to be met.
$(-k/2) = -1/(-1/3) = 3$

Therefore, **k = -6**.

Bob starts from the east end and Jane from the west end of a swimming pool, and both swim two lengths of the pool at constant rates. They pass each other twice, each time going in opposite directions. The first time they pass they are 20 feet from the east end, and the second time they are 18 feet from the west end. Assuming that each made an

instantaneous turn when they reached an end of the pool, how long is the pool, in feet?

Let x be the rate at which Bob swam and y be the rate of Jane. Let L be the length of the swimming pool.
First time they meet 20 ft from the East end. That means Bob finished 20 ft and Jane finished (L – 20) ft at that time. If t is the time, then we have the following equations.

$xt = 20$
$yt = (L – 20)$

In other words, $20/x = (L – 20)/y$.
$20y = (L – 20)x$

Now, they meet again 18 ft from west end. Now Bob has completed (L + 18) ft and Jane has completed (L + L – 18) = (2L – 18) ft.
Since the time to complete the distances is the same for both, we have the following equation.
$(L + 18)/x = (2L – 18)/y$
$(L + 18)y = (2L – 18)x$

Using both equations, we have the following.
$[20/(L – 20)] = (L + 18)/(2L – 18)$

Cross multiplying the terms we get:
$40L – 360 = L^2 – 2L – 360$
$L^2 – 2L = 40L$
$L^2 = 42L$
$L = 42$

Therefore, the length of the swimming pool is **42 ft**.

Problem of the month

In a school, during a break 12 students in a Math class decide to stand in a circle of radius 100 meters equidistant from each other. They start throwing a ball to each other. Each student throws the ball to the adjacent student only. At the end of the game if the ball was thrown & received by each student at least once, what is the total distance travelled by the ball (if the ball travelled between two students more than once, count the distance only once) rounded to nearest integer value?

Would you like submit your answer? Please click on the following link:
https://spreadsheets.google.com/viewform?formkey=dHR6ek5BazVnRVM3d01nbG1fNVdybXc6MA

Names of everybody who submitted correct answers will be published in the next edition!

Interested to know the solution for last column's problems? Refer to the end of this document!

For any questions or comments, please contact the team at NSFMathColumn@gmail.com

Answer to Problem of the month (Vol 1-10)

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Solution:

There are several websites that explain the concept of a Diophantine equation which is the basic idea of this problem. One such solution is available in the following link.

http://qedinsight.wordpress.com/2011/05/13/the-coconut-problem/

Who submitted correct answers?

- Akshaj Kadaveru (Fairfax VA)
- Vishal Gullapalli (Wayne NJ)
- Sangita Singh (Tampa FL)
- Siddarth Guha (Missouri City TX)
- Amit Jain (Santa Clara CA)
- shiva senthilkumar (knoxville TN)
- BHARATI DALAL (Ahmedabad India)
- Shraeya Madhu (Clarksburg MD)
- Sushovan Guha (Missouri City TX)
- Anand Sharma (Portland OR)
- VIDHYA KANNAN (TAMPA FL)
- Viknesh Baskar (Rochester NY)
- Krishna Bharathala (Fremont CA)
- Nishant Chittari (New Albany OH)
- Vijay Gupta (Cary NC)
- Anna Nixon (Portland OR)
- Sundar Sankaran (Voorhees NJ)
- Tanushree Pal (Ventura CA)
- Meena Shankar (Bridgewater NJ)
- Himanvi Kopuri (Denver CO)
- Saigautam Bonam (Virginia USA)
- Roshan George (charlottesville VA)
- Sreenu Pamidi (Shrewsbury MA)
- Rajasekhar Kothuri (Cupertino CA)
- ARCHANA CHAUDHARI (CARY NC)
- Anita Virjala (Santa Clara CA)
- Vinita Cheeepurupalli (Columbia SC)
- Amirtha Kasturi (Cupertino CA)
- Neha Seshadri (Novi MI)
- Yadunandan Pillai (Greenville SC)
- Shalini Dangi Mission Viejo
- Anju Garg (East Brunswick NJ)
- pillai nirjara (greenville SC)
- Thushar Mahesh (Tampa FL)
- Shashank Mahesh (Tampa FL)
- Anurag Dhawan (San Jose CA)
- Gayathri Srirajan Waukegan
- Monal Garg (East brunswick NJ)
- Shobha Dasari (Pearland TX)
- praneeth prathi (shrewsbury MA)
- Meghana Annambhotla (Southbury CT)
- Dhivya Senthil Murugan (Denver CO)
- Shiny Antony (Pembroke Pines FL)
- Desigamoorthy Nainar (Champaign IL)
- Keerti Vajrala (Aurora CO)
- yash chandak (dallas TX)
- Anusha Vajrala (Aurora CO)
- arvind nachiappan (great falls VA)
- Sunita Upadhyayula (Plainfield IL)
- Anup Hiremath (Fremont CA)
- vishal purohitham (Sanford FL)
- Samhitha Somavarapu (Manassas VA)
- simoni maniar grapevine
- rekha chandak (dallas TX)
- Pranay Malempati (Newark DE)
- Bhavana Muppavarapu (Buffalo Grove IL)
- Sonali Razdan MA
- Anitha Ramakodi (Parsippany NJ)
- trinadha muppala (overland park KS)
- Bindu Seshadri (Florida USA)
- RAMA DEVI Kodali (Cary NC)
- Sushil Upadhyayula (Plainfield IL)
- Pranav Upadhyayula (Plainfield IL)
- Abhishek Allamsetty (Herndon VA)
- anusha allamsetty (Herndon VA)
- Shreya Bellur (Dunlap IL)
- Vamsi Subraveti (Nashville TN)
- Mahant Malempati (Johns Creek GA)
- Shreyaa Raghavan (Sharon MA)
- Suchismita Nandi (Cary NC)
- Akshara Kannan (Tampa FL)
- Indumathi Prakash (Sharon MA)

- Shrutika Kumareshan (Sharon MA)
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- Nikhil Varma Pandeti (Weymouth MA)
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- Sanjana Kothuri (Cupertino CA)
- Janani Sridhar (Cupertino CA)
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- Jay Gurrala (San antonio TX)
- Vaishnavi Ravipati (Newburgh IN)
- Rahul Madala C
- Varun Singh (Tampa FL)
- Anish Madala (Chantilly VA)
- Kishna Mahankali (Bridgewater NJ)
- Manisha Solipuram (Novi MI)
- Kannan nagarajan (weston FL)
- Lata T (Phoenix AZ)
- Mrugank Gandhi (Aurora IL)
- komal handoo (tampa FL)
- Rahul Kak (Ann Arbor MI)
- Sneha Daulatani (Dix Hills NY)
- Minni Sharma (Atlanta GA)
- Lalitha Chamarajan (Mount Laurel NJ)
- Geetanjali Khanna (Piscataway NJ)
- Yash Somaiya (Fairfax VA)
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- Anirudh Chowlur Chowlur (Tamarac FL)
- Sahana Aiyer (Herndon VA)
- Jayashree Vedamurthy (Dayton NJ)
- Hemanth Chitti (Bangalore India)
- Sameer Lal (Macungie PA)
- Ashwin Vangipuram (Novi MI)
- Ritwik Pavan (Cary NC)
- Risheet Nair (Marlton NJ)
- Sayuj Shajith (Suwanee GA)
- Shruthi Snanthanam (Suwanee GA)
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- Abirami Natarajan (Plainville MA)
- pavani samala (west chester PA)
- Ajit Kadaveru (Fairfax VA)
- sreekar chitti (bangalore India)
- Ashish Kumbhardare (West Chester PA)
- Suganth Kannan US
- anmol nigam (hawthorn woods IL)
- Shritha Gunturu (Denver CO)
- preetha saravanan (Denver CO)
- Keshav Mallidi (Roanoke VA)
- A Thakkar (Naperville IL)
- Ramanan Srirajan (Waukegan IL)
- Anjali Nambrath (Marlboro NJ)
- Mythri Challa (Coralville IA)
- Janine Albert (Windsor Locks CT)
- Nihar Vallem Aurora
- srivasudha ramanujam (Hanover Park IL)
- Satvik Kolluri (Austin TX)
- Ankit Bhatia (San Diego CA)
- Esha Shakthy (Suwanee GA)
- shilpi veludandi veludandi spring
- Shreya Ramineni (Buford GA)
- Neha Chellury (Rochester Hills MI)
- Tarang Saluja (Nashua NH)
- Savan Kumar (lawrence MA)
- Sraavya Pinjala UT
- Anirudh Kuchibhatla (Hyderabad India)
- Ravikumar P (Milwaukee WI)
- Aaditya Singh (Mclean VA)
- Meghana Gudavalli (Franklin Park NJ)
- Nithin Gudavalli (Franklin Park NJ)
- Sharmila sundar (Troy OH)
- Sanjana Bhavana Akula (Morganville NJ)
- Deepankar Gupta (Naperville IL)
- Sreeniketh Vogoti (mooresville NC)
- Neela Ramesh (Cincinnati OH)
- Priyanka Kadoor (Atlanta GA)
- Lohith Annumala (Stamford CT)
- Kaushik Manchikanti (Chester Springs PA)
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- Akshay Venkat (Leawood KS)
- Ashwath Raj (San Diego CA)
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- Abhiram Akella (Lilburn GA)
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- Ashruth Suryanarayanan (Olathe KS)
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- Meera Duggal (Oak Brook, IL)
- Rashmika Kondapalli (Overland park, KS)

Thanks to all who attempted to solve the problem of the month. The Math Column team is looking forward to your continued interest and increased participation.