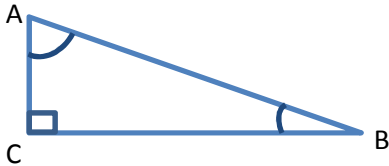


NSF Math Column – Volume 10



Triangles

In our last column, we looked at some of the trigonometrical ratios and also the Pythagoras Theorem. In this section, we'll continue to look at couple of important right triangle properties and area of triangles.



Shown above is a right triangle. Angle C is 90° and angles A and B add up to 90° . Especially, when the angles A and B are 30° and 60° or 45° and 45° , there is a special relation between the sides of the triangle. These are typically referred to as 30-60-90 and 45-45-90 triangles. For a 30-60-90 triangle, the sides are always in the ratio $1:2:\sqrt{3}$ and for 45-45-90 triangle the sides are always in the ratio $1:1:\sqrt{2}$.

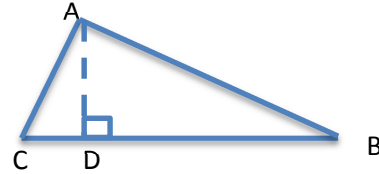
This can easily be verified using our trigonometrical ratios we learnt in the previous column. Let's say the triangle shown above is a 30-60-90 triangle. In other words, $C = 90^\circ$, $A = 60^\circ$, and $B = 30^\circ$. $\sin(B) = AC/AB$. $\sin(30^\circ) = \frac{1}{2}$. Thus AC and AB are in the ratio $1:2$. Similarly, $\tan(B) = AC/BC$. Since $\tan(30^\circ) = 1/\sqrt{3}$, AC to BC ratio $1:\sqrt{3}$. Thus, for a 30-60-90 triangle the ratio of their sides is $1:2:\sqrt{3}$.

Area of a triangle and Heron's Formula

Another widely used concept is the area of a triangle. Area of a triangle is given by the following formula.

$$\text{Area} = \frac{1}{2} * (\text{base}) * (\text{height})$$

In the triangle shown above, it will be $\frac{1}{2} * BC * AC$. This is true for any triangle and not just right triangles.



For example, in the triangle shown above, area = $\frac{1}{2} * (BC) * (AD)$

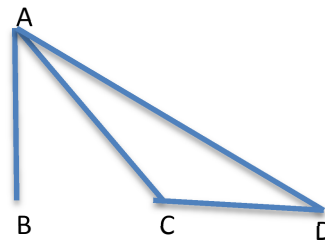
Lastly, there is another important formula to calculate area of triangle when we know the lengths of all three sides. This is called "Heron's Formula" and is a very important one.

According to this formula, the area of a triangle is given by the following:

$$\text{Area} = [s * (s - a) * (s - b) * (s - c)]^{1/2}$$

Where s is the sum of the lengths divided by 2 and a, b, c are the lengths. Let's take a look at an example.

John and Jason stand 15 ft and 20 ft from the top of a street light pole. If they are 7 ft apart what is the height of the street light pole?



Above diagram pictorially represents the problem description. AB is the street light pole. C is the location where John is standing and D is where Jason is standing. We also know that $AC = 15$, $AD = 20$, and $CD = 7$.

$$s = (15 + 20 + 7)/2 = 21$$

NSF Math Column – Volume 10

$$\begin{aligned} \text{Using Heron's formula, area of the triangle} &= \\ [21*(21 - 15)*(21 - 20)*(21 - 7)]^{1/2} &= \\ = [21*6*1*14]^{1/2} &= \\ = 42 \text{ ft}^2 & \end{aligned}$$

Now, based on our previous formula:

$$\begin{aligned} \text{Area} &= \frac{1}{2} * (\text{base}) * (\text{height}) \\ &= \frac{1}{2} * (\text{CD}) * (\text{AB}) \end{aligned}$$

$$\text{Therefore } \text{AB} = 2 * (\text{Area}) / (\text{CD}) = 2 * (42) / 7.$$

Height of the street light pole = 12 ft.



Competitive Math

(★ indicates difficulty level)

Ryan got 80% of the problems on a 25-problem test, 90% on a 40-problem test, and 70% on a 10-problem test. What percent of all problems did Ryan answer correctly? ★

Important formula to remember for percentage problems is:

$$\text{Part} = (\text{Percentage}/100) * \text{Whole}$$

In the first test, he got 80% of the 25 problems, which is $(80/100)*25 = 20$. In the second test, he got $(90/100)*40 = 36$. In the last one, he got $(70/100)*10 = 7$.

Overall, he got $20 + 36 + 7 = 63$ problems out of $25 + 40 + 10 = 75$ problems. Therefore, we can use the above formula again to calculate the % of correct problems in all three tests combined as follows.

$$(63/75) * 100 = \mathbf{84\%}$$

If $4^x - 4^{x-1} = 24$, what is the value of $(2x)^x$? ★★

This problem applies basic rules of exponents. Remember, in an exponent of the form a^b , a is

referred to as the “base” and b is referred to as the “exponent”. An important rule to remember here is:

$$a^{b-c} = a^b \times a^{-c} = a^b/a^c$$

Applying this to our equation, we have:

$$4^x - 4^x/4 = 24$$

$$4^x(1 - \frac{1}{4}) = 24$$

$$4^x(3/4) = 24$$

$$4^x = 32$$

$$(2^2)^x = 32$$

Another rule to remember when dealing with exponents is:

$$(a^b)^c = a^{bc}$$

Hence we get $2^{2x} = 32$. Therefore $2x = 5$ and $x = 5/2$.

Now using this values of x to find $(2x)^x$, we have:

$$(2(5/2))^{5/2}$$

$$= 5^{5/2}$$

$$= \mathbf{25\sqrt{5}}$$

In a school with 400 eighth graders, 117 take algebra, 109 take advanced computer science (ASC), and 114 take industrial technology (IT). 70 students sign up for both algebra and ASC, 34 sign up for both algebra and IT, and 29 sign up for both ASC and IT. If 164 students take none of these courses, how many signed up for all three courses? ★★★

Let's assume X to be the number of students who took all three subjects. Note that these X students are counted multiple times in various categories. Let's analyze that.

Given that 70 students signed up for both Algebra and ASC, actual number of students signed up for these subjects only are $70 - X$.

Similarly, actual number of students signed up

NSF Math Column – Volume 10

for Algebra and IT only are $34 - X$. Number of students signed up for IT and ASC only are $29 - X$.

Students signed up for Algebra only = $117 - (70 - X) - X - (34 - X) = 13 + X$.

Students signed up for ASC only = $109 - (70 - X) - X - (29 - X) = 10 + X$.

Students signed up for IT only = $114 - (34 - X) - X - (29 - X) = 51 + X$.

Given that 164 students did not sign up for any of the three subjects, total number signed up for one of more these subjects = $400 - 164 = 236$.

Now we can add up all our numbers and should equal 236.

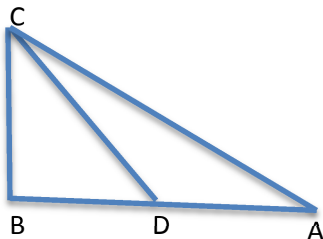
$$(13 + X) + (10 + X) + (51 + X) + (70 - X) + (34 - X) + (29 - X) + X = 236$$

$$207 + X = 236$$

$$X = 29$$

Therefore, 29 students took all three subjects. The same can be visualized with a Venn diagram as well.

Suppose $\angle ABC = 90^\circ$, $\angle CDB = 45^\circ$, $\angle CAB = 30^\circ$, and $AD = 2$. Find the length of BC . ✨



Let's make use of the trigonometric ratios we reviewed in the last column. Let's set $BC = X$.

$\tan(D) = \text{opposite side}/\text{adjacent side} = BC/BD$
Given angle $D = 45^\circ$ and also $\tan(45^\circ) = 1$, we have $BC = BD = X$.

Similarly, $\tan(A) = BC/AB$. Given that the angle $A = 30^\circ$ and $\tan(30^\circ) = 1/\sqrt{3}$, we have $BC = AB/\sqrt{3}$

$$X = (BD + AD)/\sqrt{3}$$

$$X = (X + 2)/\sqrt{3}$$

$$(\sqrt{3} - 1)X = 2$$

$$X = 2/(\sqrt{3} - 1)$$

If the ratio of $2x - y$ to $x + y$ is 2:3, what is the value of the ratio x to y ? ✨

Given that $(2x - y)/(x + y) = 2/3$.

Rewriting the equation, we have the following.

$$3(2x - y) = 2(x + y)$$

$$6x - 3y = 2x + 2y$$

$$6x - 2x = 2y + 3y$$

$$3x = 5y$$

$$x/y = 5/3$$

In a raffle 20 tickets are sold. Two prizes will be given. If a student buys two tickets, what is the probability that this student wins at least one prize? ✨ ✨

Let's calculate the total number of ways 2 tickets can be bought out of the available 20 tickets. It is given by $20!/(2! * (20 - 2)!)$. In other words, there are 190 ways to buy the 2 tickets.

Since there are two winning cards, 18 others are the losing cards. Student can pick 2 cards from those 18 cards in $18!/(2!16!) = 153$ ways. Therefore, there are $190 - 153 = 37$ ways to win.

In order to calculate the probability, we basically divide the number of favorable outcome by total outcomes.

$$\text{Probability} = 37/190.$$

NSF Math Column – Volume 10



Problem of the month

Five sailors decide to spend a night on an island and see a pile of coconuts near their resting place. They decide to divide the coconuts equally among themselves the next morning. During the night, one of them wakes up and decides to take his share. He realizes that the division will not be even and hence, throws one coconut to a monkey to make the division even and takes his share and goes back to sleep. The other four sailors do likewise, one after the other, each throwing a coconut to the monkey and taking a fifth of the remaining pile of coconuts. In the morning, the five sailors realize that they have to throw a coconut to the monkey in order for them to divide the coconuts equally among them. What is the smallest number of coconuts that must have been in the original pile?

Would you like submit your answer? Please click on the following link:

<https://spreadsheets.google.com/viewform?formkey=dHR6ek5BazVnRVM3d01nbG1fNVdybXc6MA>

Names of everybody who submitted correct answers will be published in the next edition!



Interested to know the solution for last column's problems? Refer to the end of this document!

For any questions or comments, please contact the team at NSFMathColumn@gmail.com

NSF Math Column – Volume 10

Answer to Problem of the month (Vol 1-9)

No

Solution:

We are asked whether the sum of first 100 factorials is divisible by 8. Let's see if there is a pattern in the factorial values.

1! = 1	Sum = 1
2! = 2	Sum = 3
3! = 6	Sum = 9
4! = 24	Sum = 33
5! = 120	Sum = 153
6! = 720	Sum = 873
7! = 5040	Sum = 5913
8! = 40320	Sum = 46233
9! = 362880	Sum = 409113
10! = 3628800	Sum = 4037913
11! = 39916800	Sum = 43954713
12! = 479001600	Sum = 522956313
13! = 6227020800	Sum = 6749977113
14! = 87178291200	Sum = 93928268313
15! = 1307674368000	

Now, starting 15!, last 3 digits will always be zero and the sum will have 313 as the last three digits as well. Based on divisibility rules for 8, if last 3 digits of the number is divisible by 8, then the entire number is divisible. Since 313 is not divisible by 8, we can conclude that the sum of first 100 factorials is not divisible by 8.



Who submitted correct answers?

- Sreedhar, Atluri (Weston, FL)
- Anita, Patel (Princeton, NJ)
- Abirami, Natarajan (Plainville, MA)
- Sumathi, Ganjam (Monroe twp, NJ)
- Tanushree, Pal (Ventura, CA)
- BHARATI, DALAL (Ahmedabad, India)
- Desigamoorthy, Nainar (Champaign, IL)
- Sanjana, Challa (USA)
- Shalini, Dang (Mission Viejo)
- Nitin, Sadalgekar (East Windsor, NJ)
- Akshaj, Kadaveru (Fairfax, VA)
- preetha, saravanan (denver, CO)
- Pranav, Rekapalli (Atlanta, GA)
- Sameer, Lal (Macungie, PA)
- Saisuki, Putumbaka (Basking Ridge, NJ)
- Akash, Karanam (Sugar Land, TX)
- Siddarth, Guha (Missouri City, TX)
- Shivani, Guha (Missouri City, TX)
- Deepankar, Gupta (Naperville, IL)
- Thushar, Mahesh (Tampa, FL)
- Shashank, Mahesh (Tampa, FL)
- Adhith, Palla (Hoffman Estates, IL)
- Ravikiran, Komirisetty (Irvine, CA)
- Sriraj, Atluri (Weston, FL)
- Shwetark, Patel (Herndon, VA)
- Ashwath, Raj (San Diego, CA)
- Rishita, Saladi (Campbell, CA)
- Tanishq, Kancharla (Middlebury, CT)
- Simoni, Maniar (Grapevine, TX)
- Anup, Hiremath (Fremont, CA)
- Anju, Garg (east brunswick, NJ)
- sreekar, chitti (bangalore, india)
- Hemanth, Chitti (Bangalore, India)
- Kannan, nagarajan (weston, FL)
- Viknesh, Baskar (Rochester, NY)
- Jai, Kannan (Livonia, MI)
- Shaila, Patankar (Marlboro, NJ)
- komal, handoo (tampa, FL)
- Rahul, Madala (Chantilly, VA)
- Anish, Madala (Chantilly, VA)
- Roshan, George (charlottesville, VA)
- Abhishek, Allamsetty (Herndon, VA)
- Anusha, Allamsetty (herndon, VA)
- Sonali, Razdan (MA)
- Bharat, Kabra (edison, NJ)
- Dhivya, Senthil Murugan (Denver, CO)
- Ankit, Patel (Princeton, NJ)
- Himanvi, Kopuri (Denver, CO)
- Sreenu, Pamidi (Shrewsbury, MA)
- Anika, Ramachandran (cupertino, CA)
- shalini, Agarwal (Overland Park, KS)
- Vibha, Agarwal (Overland Park, KS)
- Sudharani, Tangirala (Campbell, CA)

NSF Math Column – Volume 10

- Janani,Sridhar (Cupertino, CA)
- Nina,Vendhan (Denver, CO)
- Pujita,Tangirala (Campbell, CA)
- Rohin,Tangirala (Campbell, CA)
- Neha,Dudipala (Columbus, OH)
- Sneha,Prasath (Westford, MA)
- Venkatesh,Madapoosi (St. louis, MO)
- Shaheel,Mitra (Cincinnati, OH)
- Vamsi,Subraveti (Nashville, TN)
- Uma,Kothuri (Naperville, IL)
- Suveer,Kothuri (Naperville, IL)
- Akshay,Prabhushankar (Olathe, KS)
- Dhruv,Puri (Austin, TX)
- Anika,Puri (Austin, TX)
- Dhruva,Mambapoor (Austin, TX)
- samhitha,somavarapu (Manassas, VA)
- Shraeya,Madhu (Clarsburg, MD)
- Sraavya,Pinjala (Salt Lake City, UT)
- Vishal,Purohitham (Sanford, FL)
- Sahana,Aiyer (Herndon, VA)
- Navya,Prabhushankar (Olathe, KS)
- preetham,bachina (pleasanton, CA)
- Ramesh,Madesiah (Johns Creek, GA)
- Pranav,Kotilingam (North Bruswick, NJ)
- Satvik,Reddy (Jacksonville, FL)
- Sayuj,Shajith (Suwanee, GA)
- Sneha ,Reddy (Jacksonville, FL)
- Sayak,Chatterjee (Winchester, MA)
- Gargi,Sadalgekar (East Windsor, NJ)
- Gauthum,Srinivas (Portland, OR)
- Meghana,N (Austin, TX)
- Anna,Nixon (Portland, OR)
- Sindhuja,Karanam (Sugar Land, TX)
- Anita,Virjala (Santa Clara, CA)
- Chitra,Balasubramaniam (Columbus, OH)
- Ram,Tyagi (Chicago, IL)
- Anitha,Ramakodi (Parsippany, NJ)
- Bindu,Seshadri (Miami, FL)
- Amirtha,Kasturi (CA)
- Arvind ,Chandaka (hillsborough, NJ)
- Asha,Chandaka (Hillsborough, NJ)
- VAMSI,CHAVALI (OVERLAND PARK, KS)
- Nishant,Chittari (New Albany, OH)
- Akshay,Venkat (Leawood, KS)
- Alor,Sahoo (Bakersfield)
- Mahati,Malladi (fairfax, VA)
- Anirudh ,Chowalur (tammarac, FL)
- Varun,Singh (Seffner, FL)
- Rishheet,Nair (Marlton, NJ)
- Shreyaa,Raghavan (Sharon, MA)
- Bhavana ,Muppavarapu (Buffalo Grove, IL)
- praneeth,prathi (shrewsbury, MA)
- Jayanth,Gunda (Hyderabad, India)
- Suma,Maineni (Chicago, IL)
- Shriitha,Gunturu (Aurora, CO)
- Anjali,Nambrath (Marlboro, NJ)
- Shivani,Senguttuvan (Champaign, IL)
- Keerti,Vajrala (Aurora, CO)
- Rinni, Bhansali (Melville, NY)
- Akaash,Sanyal (Cary, NC)
- Nithik,Balachandran (Alpharetta, GA)
- Anurag,Vuthunuri (South Elgin, IL)
- Ankit,Bhatia (San Diego, CA)
- Aditya,Chittari (New Albany, OH)
- Arya,Goutam (Hillsborough, CA)
- Pratab,Narayanaswamy (Marietta, GA)
- Aneesh ,Agarwal (Cary, NC)
- Ashwin,Vangipuram (Novi, MI)
- Shwetha,Mudalegundi (Cunning, GA)
- Joshi,Abain (Miami, FL)
- Amogh,Thalanki (Wellesley)
- Yash,Somaiya (VA)
- Akhil Aggarwal,Aggarwal (Buffalo Grove, IL)
- Sahana,Kumar (San Diego, CA)
- Ramesh,Avva (Huntersville, NC)
- Maya,Shankar (Bridgewater, NJ)
- Mukta,Valluru (Austin, TX)
- ilakiya,udhayakumar (metuchen, NJ)
- Sankar,Mahadevan (Dayton, NJ)
- Mridula,Gopal (Chennai, India)
- Mrugank,Gandhi (Aurora, IL)
- megha,shukla (edison, NJ)
- Arnav,Singh (Tampa, FL)
- Arvind,Chava (Herndon, VA)
- Shraman,Sen (Morrisville, NC)
- Sneha,Daulatani (Dix Hills, NY)
- sharmila,sundar (troy, OH)
- Shopnavo,Biswas (Fremont, CA)
- Saujas,Nandi (Cary, NC)
- Shravan,Ravishankar (Louisville, KY)
- Gayathri,Srirajan (Waukegan, IL)

NSF Math Column – Volume 10

- Meghana,Annambhotla (Southbury, CT)
- rekha,chandak (dallas, TX)
- yash,chandak (dallas, TX)
- Sarat,Sagaram (Aurora, IL)
- Shiny,Antony (Pembroke Pines, FL)
- Ranjan,Veludandi,spring
- Gayatri,Ramesh (Cincinnati, OH)
- Chetana,Ramesh (Cincinnati, OH)
- Meena,Shankar (Bridgewater, NJ)
- Tarang,Saluja (Nashua, NH)
- Rao,Allu (St. louis, MO)
- Rahul,Kak (Chelsea, MI)
- Anusha,Vajrala (Aurora, CO)
- Rohit,Gangupantulu (Whitehall, PA)
- Leela,Pakanati (Dunlap, IL)
- Janine,Albert (Windsor Locks, CT)
- Indumathi,Prakash (Sharon, MA)
- Sangeetha,Kamidi (Coppell, TX)
- Shreya,Ramineni (Buford, GA)
- Varun,Chheda (Indianapolis, IN)
- Reena,Mallannagari (Madison, WI)
- Aayush,Singh (Seffner, FL)
- Anirudh ,Kuchibhatla (Hyderabad, India)
- Mani,Jithendra (Johnston, IA)
- Manju,Arasaiah (Chandler, AZ)
- Kavita ,Daga (Cleveland, OH)
- Manisha,Solipuram (Novi, MI)
- Chandrasekhar,Sundar Rajan (Irving, TX)
- Rajasekhar,Kothuri (Cupertino, CA)
- Laasya,Renganathan (Cary, NC)
- Vishik,Bhalla (Nashua, NH)
- YN Sharma,Kuchibhatla (Vijayawada, India)
- Nandini,Iyer (Placentia, CA)
- shrutika,Kumareshan (sharon, MA)
- Ananya,Yammanuru (St charles, IL)
- Srivasudha,Ramanujam,Hanover Park
- Anirudh,Rangaswamy (Centerville, OH)
- Anish,Bose (Austin, TX)
- Bhargav,Mallidi (Roanoke, VA)
- Shreya,Bellur (Dunlap, IL)
- Alekya,Vejendla (Houston, TX)
- syamala,malladi (fairfax, VA)
- Aayush,Gupta (Edison, NJ)
- Ishaan,Gupta (Edison, NJ)
- Rohin,Maganti (TX)
- neeyanth,kopparapu (herndon, VA)
- Kavya,Kopparapu (Herndon, VA)
- Riya,Dabbi (herndon, VA)
- Abhraneel,Dutta (Cary, NC)
- Samiksha,Mulpuri (Austin, TX)
- PRANEET,DARA (PALATINE, IL)
- Nanda ,Ravipati (Newburgh, IN)
- Nymisha,Mattapalli (Herndon, VA)
- Rachana,Madhukara (San Diego, CA)
- Sreeniketh,Vogoti (Mooresville)
- Nisha,Goel (Woburn, MA)
- Geetanjali,Khanna (Piscataway, NJ)
- Sanjna ,Khanna (Piscataway, NJ)
- Shruthi,Santhanam (Suwanee, GA)
- shiva,senthilkumar (knoxville, TN)
- Yadunandan,Pillai (Greenville, SC)
- Chaitu,Konjeti (Nashville, TN)
- Saigautam ,Bonam (VA)
- Vivek,VAllurupalli (Wildwood, MO)
- Navya,Ravavarapu (Fremont, CA)
- Vanitha,Sankaranarayanan (Walnut Creek, CA)
- vinay,ramesh (Bangalore, India)
- subhash,siripurapu (san antonio, TX)
- sri,bhamidipati (farmington hills, MI)
- Sanjana,M (Herndon, VA)
- Rohan,Chakraborty (Roanoke, VA)
- vidhya,kannan (Riverview, FL)
- Veeresh ,Nama (Auburn Hills, MI)
- Manoj,Dixit (Appleton, WI)
- Ashish,Kumbhardare (West Chester, PA)
- Eesha,Deepak (San Jose, CA)
- Divya ,Gubba (Flower Mound, TX)
- Teevyah,Yuva Raju (Folsom, CA)
- Rama,Kodali (Cary, NC)
- Shreeman,Nimmagadda (Ann Arbor, MI)
- Pratiik,Kaushik (Pittsburgh, PA)
- Nihar ,Vallem (Aurora, IL)
- Satvik,Pendyala (irving, TX)
- Vinita,Cheepurupalli (Columbia, SC)
- Ajit,Kadaveru (Fairfax, VA)
- Kiran,Nariseti (Columbus, OH)
- shankarganesh,krishnamurthy (columbus, OH)
- Siva,Ranganeni (Middlebury, CT)
- Mahima,Parupalli (Tampa, FL)
- Aditya,Vargheese (Overland Park, KS)

NSF Math Column – Volume 10

- Jash,Patel (Fort Myers, FL)
- Kartik ,Dhinakaran (Westlake, OH)
- Amogh,Gaitonde (CA)
- rahul,maini (peoria, IL)
- Nithya,J (Cumming, GA)
- Aravind,Kuchibhatla (Hyderabad, India)
- Priyanka,Roy (Framingham, MA)
- Ravikumar,Ponnusamy (Milwaukee, WI)
- Sanjana,Sastry (Edison, NJ)
- Vishal,Gullapalli (Wayne, NJ)
- Mahant,Malempati (Johns Creek, GA)
- Amit,Anand (NJ)
- Shreya,Shankar (Bridgewater, NJ)
- Surya,Jaladi (St.louis, MO)
- Kavya,Ravichandran,OH
- adit,a (TX)
- Anudeep,Udumula (Bear, DE)
- Krishna,Bharthala (Fremont, CA)
- Soumika,Guduru (San Diego, CA)
- Pranav,Upadhyayula (Plainfield, IL)
- AARUSH,PRASAD (TAMPA, FL)
- Monal,Garg (East Brunswick, NJ)
- Sanjana,Kothuri (Cupertino, CA)
- Esha ,Shakthy (Suwanee, GA)

Thanks to all who attempted to solve the problem of the month. The Math Column team is looking forward to your continued interest and increased participation.