

NSF Math Column



Problem of the month

How many natural numbers require 3 digits when written in base 12, but require 4 digits when written in base 8?

Would you like submit your answer? Please click on the following link:

<https://spreadsheets.google.com/viewform?formkey=dHR6ek5BazVnRVM3d01nbG1fNVdybXc6MA>

Names of everybody who submitted correct answers will be published in the next edition!



Interested to know the solution for last column's problems? Refer to the end of this document!

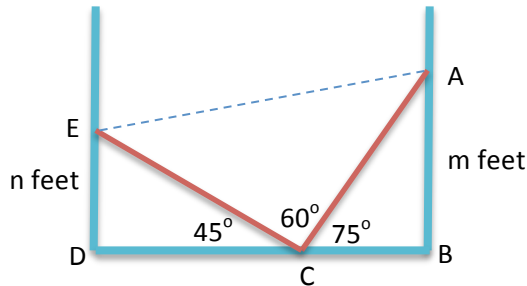
For any questions or comments, please contact the team at NSFMathColumn@gmail.com

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Answer to Problem of the month (Vol 2-8)

m feet

Solution (Source: VTS):



Let's say the ladder initially touches the wall at point A. Given, $AB = m$ feet and angle $ACB = 75^\circ$. When the ladder leans on the opposite wall, let's say it touches the wall at E. It is given that $ED = n$ feet and angle $ECD = 45^\circ$. Based on this, we can derive the angle $ACE = 60^\circ$. Since $\triangle DEC$ is $45^\circ-90^\circ-45^\circ$, length $EC = n\sqrt{2}$ and $DC = n$.

In the $\triangle ABC$, let's use the following trigonometric function.

$$\cos(A) = \text{adjacent/hypotenuse}$$

$$\cos(15^\circ) = m/AC$$

$$\text{Also, } \cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B).$$

$$\cos(15^\circ) = \cos(45^\circ - 30^\circ) = (1/\sqrt{2})(\sqrt{3}/2) + (1/\sqrt{2})(1/2) = (\sqrt{3} + 1)/2\sqrt{2} = m/AC$$

$$\text{Hence, } AC = (2m\sqrt{2})/(\sqrt{3} + 1).$$

Let's find BC now using the tan function.

$$\tan(A) = \text{Opposite/Adjacent} = BC/m$$

$$\tan(A - B) = (\tan(A) - \tan(B))/(1 + \tan(A)\tan(B))$$

$$\tan(15^\circ) = \tan(45^\circ - 30^\circ)$$

$$= [\tan(45^\circ) - \tan(30^\circ)]/[1 + \tan(45^\circ)\tan(30^\circ)]$$

$$= [1 - (1/\sqrt{3})]/[1 + (1/\sqrt{3})]$$

$$BC = m(\sqrt{3} - 1)/(\sqrt{3} + 1)$$

Now, consider the triangle ABC. It is an equilateral triangle.

$$EC = AC = AE$$

$$EC = AC \Rightarrow n\sqrt{2} = (2m\sqrt{2})/(\sqrt{3} + 1)$$

$$n = (2m)/(\sqrt{3} + 1)$$

$$\text{Finally, } BD = BC + DC = m(\sqrt{3} - 1)/(\sqrt{3} + 1) + n$$

$$= m(\sqrt{3} - 1)/(\sqrt{3} + 1) + (2m)/(\sqrt{3} + 1)$$

$$= m$$

Length between the two walls = **m feet**.



Who submitted correct answers?

- Arvind Subramanian (Olathe, KS)
- Sankar Mahadevan (Dayton, NJ)
- Ananth Kumar (Pleasanton, CA)
- Vivek Gaddam (Westford, MA)
- Shravani Samala (West Chester, PA)
- Aneesh Agarwal (NC)
- sri jaladi (St. Louis, MO)
- Desigamoorthy Nainar (Champaign, IL)
- Anjali Nambrath (Marlboro, NJ)
- Siddarth Guha (Missouri City, TX)
- Sushovan Guha (Missouri City, TX)
- Pranav Kavikondala (Plano, TX)
- Vinu Kumar (Kernersville, NC)
- Anish Chaluvadi (Simpsonville, SC)
- Sreekar Chitti (Bangalore, India)
- Hemanth Chitti (Bangalore, India)
- Rinni Bhansali (NY)
- jai kannan (Livonia, MI)
- Harish Jaisankar (Livonia, MI)

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- Rsheet Nair (Marlton, NJ)
- Rama devi kodali (Cary, NC)
- Rohith Sai Edupuganti (Cary, NC)
- Anirudh Kuchibhatla (Hyderabad, India)
- Srivani Edupuganti (Cary, NC)
- Anudeep Udumula (Bear, DE)
- Snigdha Allaparthi (Westborough, MA)
- Gita Balakrishnan (Sunnyvale, CA)
- Gaurav Hardykar (Princeton Junction)
- Venkatesh Madapoosi (GROVER, MO)
- Meena Shankar (Bridgewater, NJ)
- suraj rathi (Edison, NJ)
- Dhivya Senthil Murugan (Dever, CO)
- Savan Kumar (Lawrence)
- Neha Khandelwal (FAIRFAX)
- Pranav Upadhyayula (Plainfield, IL)
- Preetham Bachina (Pleasanton, CA)

Thanks to all who attempted to solve the problem of the month. We look forward to your continued interest and increased participation!