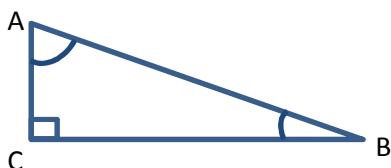


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Triangles

In our last column, we looked at an interesting problem in the “Problem of the month” section. This basically involved the application of **Pythagorean Theorem**. In this section, we’ll review the concept and a few useful ratios. The most obvious fact about triangles is that it is a 3-sided polygon.



In the above triangle, angle C is 90° . A triangle where one of the angles is 90° (i.e. right-angle) is often referred to as “right-angled triangle”. In addition, sum of interior angles A, B, and C is 180° . In fact, sum of interior angles of any n-sided polygon is given as $(n - 2) * 180^\circ$.

Let’s take a look at the Pythagoras Theorem. It is applicable only for right-angled triangles and states that the square of hypotenuse is equal to the sum of squares of the other two sides. In the triangle shown above, AB (longer side) is the hypotenuse and according to the theorem, we have the following.

$$AB^2 = AC^2 + BC^2$$

This can be easily verified by constructing squares for each side of the right-angled triangle and validating that the area of the square on the hypotenuse is same as the sum of the areas of squares on the other two sides.

Let’s review few key facts about the sides of the triangle and internal angles. These involve ratios involving the hypotenuse, adjacent side and the opposite side.

In our example, for angle B, AC is the opposite side and BC is the adjacent side.

Ratio of opposite side to the hypotenuse is called the sine of an angle written as “Sin(angle)”.

$$\text{Sin}(B) = AC/AB$$

Similarly, the ratio of adjacent side to the hypotenuse is called the cosine of an angle written as “Cos(angle)”.

$$\text{Cos}(B) = BC/AB$$

Following are some other important ratios.

Tangent of an angle: $\text{Tan}(B) = AC/BC$

Cotangent of an angle: $\text{Cot}(B) = BC/AC$

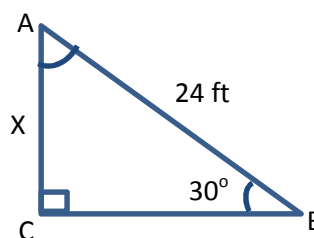
Secant of an angle: $\text{Sec}(B) = AB/BC$

Cosecant of an angle: $\text{Cosec}(B) = AB/AC$

These six ratios are often referred to as “**trigonometrical ratios**”.

Let’s look at an example of how these ratios can be used. A ladder 24 ft in length is used to clean a window in a building. If the ladder is positioned in such a way that it forms a 30° angle with the floor, how high is the window?

Following is a pictorial representation of the problem description.



In this problem, we know the hypotenuse and the angle B. Unknown is the opposite side. Using the Sine angle ration, we have:

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$$\sin(30) = X/24$$

$$X = 24 * \sin(30)$$

$$\sin(30) = 1/2. \text{ Hence } X = 12 \text{ ft.}$$

In other words, the window is at a height of 12 ft from the floor. This is a simple example to illustrate the use of trigonometric ratios. There are several useful applications in practice.



Competitive Math

(☆☆ indicates difficulty level)

If John gets a 97 on his next math test, his average will be 90. If he gets 73, his average will be 87. How many tests has John already taken? ☆☆☆

Approach #1:

Let's say the current average is X from N tests. Total of all scores so far is $N * X$. In the $(N + 1)^{\text{th}}$ test, if he scores 97 then the new total score is $NX + 97$ and the new average is $(NX + 97)/(N + 1)$. We are given that this average is 90.

$$(NX + 97)/(N + 1) = 90$$

$$NX + 97 = 90(N + 1)$$

$$NX + 97 = 90N + 90$$

$$90N - NX = 7$$

Now, if he gets 73 in this $(N + 1)^{\text{th}}$ test, then his new total score will be $NX + 73$ and the new average will be $(NX + 73)/(N + 1)$. This average is given as 87.

$$(NX + 73)/(N + 1) = 87$$

$$NX + 73 = 87(N + 1)$$

$$NX + 73 = 87N + 87$$

$$NX - 87N = 14$$

By solving the two equations, we get $3N = 21$ or $N = 7$.

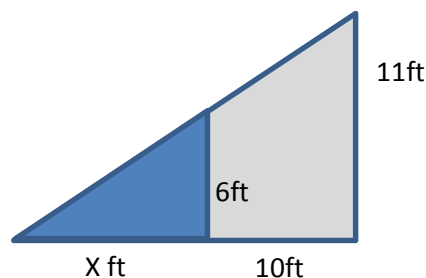
So, John already took 7 tests.

Approach #2:

Another easy way to solve this is to understand what average implies. In simple terms, average is nothing but distributing the total equally among the items involved in calculating the average. For example, if 3 students scored 20, 25, and 30 in a test then the average is 25. In other words, we calculate the total (75) and distribute equally among the three students. Using that same concept, we see that the test score being either 97 or 73 – a swing of 24 points in the last test – results in a change in average of 3 points. In other words, there are 8 tests ($24/3$) including the last test. Hence, John already took 7 tests.



A man is standing 10 ft away from a street light. The man is 6 ft tall and the light is 11 ft high. How long is the man's shadow?



Based on the diagram above, it is clear that we have a scenario of similar triangles. Triangles are similar if their corresponding (matching) angles are congruent (equal) and the ratios of their corresponding sides are in proportion. Using this similarity concept, we can write the following:

$$(11/6) = (10 + X)/X$$

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$$\begin{aligned}11X &= 6(10 + X) \\11X &= 60 + 6X \\5X &= 60 \\X &= 12\end{aligned}$$

Thus the length of the shadow is **12 ft.**



Two successive discounts of 10% have the same effect as a single discount of what %? ✨

Important formula to remember for percentage problems is:

$$\text{Part} = (\text{Percentage}/100) * \text{Whole}$$

In problems like these, it is easier to start with 100. First discount of 10% will result in 90% remaining – same as 90. Next discount of 10% on 90 will result in $(90 - 9) = 81$.

Now, if we need to accomplish the same using a single discount, then we need to give a discount of 19 out of 100 or **19%**.



A pair of fair dice is thrown. What is the probability that the two numbers that appear differ by an even number? ✨

When calculating probability, important formula to remember is:

$$\text{Probability of an event} = (\# \text{ of favorable outcomes}) / (\text{Total} \# \text{ of outcomes})$$

$$\text{Total possible outcomes} = 36$$

Let's list all the favorable outcomes.

$$(1,3), (1,5), (2,4), (2,6), (3,5), (4,6), (3,1), (5,1), (4,2), (6,2), (5,3), (6,4)$$

$$\# \text{ of favorable outcomes} = 12$$

Probability that the numbers from the two dice differ by an even number = $12/36 = 1/3$ or **33.33%**.



A tank has three independent inlet pipes, A, B, and C. A and B will fill the tank in z minutes; A and C will fill the tank in y minutes; and B and C will fill the tank in x minutes. How long will it take for pipe A alone to fill it? ✨

In order for finding the time taken by A to fill the tank, we need to find the rate of flow for pipe A. If A and B can fill the tank in z minutes, then the rate of flow for A and B together is $1/z$.

In other words, A and B can fill $(1/z)$ of the tank in one minute.

$$A + B = 1/z$$

Similarly,
 $A + C = 1/y$ and $B + C = 1/x$

Adding the first two equations, we get the following.

$$2A + B + C = (1/z) + (1/y)$$

Substituting the value for B + C from third equation, $2A + (1/x) = (1/z) + (1/y)$

$$\begin{aligned}2A &= (1/z) + (1/y) - (1/x) \\2A &= ((y + z)/yz) - (1/x) \\2A &= (xy + xz - yz)/xyz \\A &= (xy + xz - yz)/2xyz\end{aligned}$$

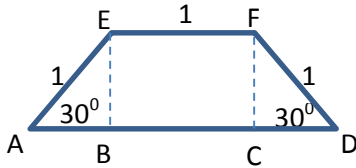
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This is the rate at which A fills the tank. Hence, the total time taken by A alone is:

$$2xyz / (xy + xz - yz) \text{ minutes.}$$



What is the area of the trapezoid below? ✨ ✨



In order to solve this, we need to find out the length of AB (same as CD) and BE (same as FC).

$$\begin{aligned} \cos(A) &= AB/AE \\ AB &= AE * \cos(A) \\ &= 1 * (\sqrt{3}/2) \end{aligned}$$

$$BE = \sqrt{AE^2 - AB^2} = \sqrt{1 - 3/4} = \sqrt{1/4} = 1/2.$$

Similarly, $CD = \sqrt{3}/2$ and $FC = 1/2$.

$$\begin{aligned} \text{Area of triangle AEB} &= (1/2) * (\sqrt{3}/2) * (1/2) \\ &= \sqrt{3}/8 \end{aligned}$$

$$\text{Area of rectangle BEFC} = 1 * (1/2) = (1/2)$$

$$\begin{aligned} \text{Total area of the trapezoid} &= 2 * (\sqrt{3}/8) + (1/2) \\ &= (\sqrt{3} + 2)/4 \end{aligned}$$



Problem of the month

One day, John and his brother Dave argue on who gets to have the only ice cream left in the refrigerator. Their dad decides to mediate and asks them to solve a puzzle. Whoever correctly solves the puzzle first will get the ice cream. The question is: "Is the sum of first 100 factorials divisible by 8?" Can you solve this puzzle?

Would you like submit your answer? Please click on the following link:

<https://spreadsheets.google.com/viewform?formkey=dHR6ek5BazVnRVM3d01nbG1fNVdVbXc6MA>

Names of everybody who submitted correct answers will be published in the next edition!



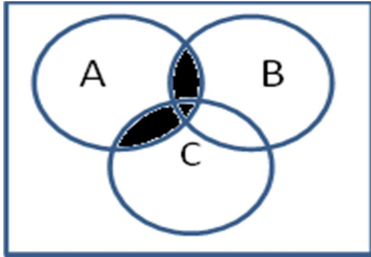
Interested to know the solution for last column's problems? Refer to the end of this document!

For any questions or comments, please contact the team at NSFMathColumn@gmail.com

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Answers to Practice Problems (Vol 1-8)

1. $15 + 12 - 8 = 19$
- 2.

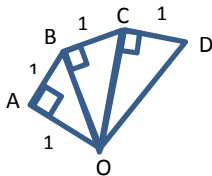


Answer to Problem of the month (Vol 1-8)

7 ft

Solution:

This is an example application of Pythagoras Theorem. Basically, we need to work backwards for this problem. Following diagram illustrates the last 3 throws.



Let O be the point from where Samir threw and A is the spot for 49th throw, B the 48th throw and so on. We are given that $OA = 1$, $AB = 1$, $BC = 1$, etc. We can find the distance of the 48th throw as follows.

$$OB = \sqrt{OA^2 + AB^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\text{Similarly, } OC \text{ (47}^{\text{th}} \text{ throw)} = \sqrt{OB^2 + BC^2} \\ = \sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3}$$

This pattern will continue and the first throw will be $\sqrt{49} = 7$ ft.



Who submitted correct answers?

- Akshaj, Kadaveru, (Fairfax, VA)
- Sruthi, Parthasarathi, (Mason, OH)
- Jay, Gurralla, (San Antonio, TX)
- shalini, Agarwal, (Overland Park, KS)
- Nitin, Sadalgekar, (East Windsor, NJ)
- Nikhil, Parchuri, (Princeton, NJ)
- Shaila, Patankar, (Marlboro, NJ)
- Vibha, Agarwal, (Overland Park, KS)
- Sneha, Reddy, (Jacksonville, FL)
- Mana, Singri, (Southlake, TX)
- Madhavi, Reddy, (Jacksonville, FL)
- Kannan, Dr. Nagarajan, (Weston, FL)
- Nina, Vendhan, (Denver, CO)
- Latha, P, (Hoffman Est, IL)
- Suganth, Kannan, (weston, FL)
- Sumukh, Shivakumar, (wesley chappel, FL)
- Adhith, Palla, (Hoffman Estates, IL)
- Sameer, Lal, (Macungie, PA)
- Krishna, Bharathala, (Fremont, CA)
- Anup, Hiremath, (Old Bridge, NJ)
- Anita, Patel, (Princeton, NJ)
- Rachana, Madhukara, (San Diego, CA)
- Anika, Ramachandran, (Cupertino, CA)
- Ankit, Patel, (Princeton, NJ)
- Tarang, Saluja, (Nashua, NH)
- sanmeshkumar, Udhayakumar, (Metuchen, NJ)
- Gargi, Sadalgekar, (East Windsor, NJ)
- Sudharani, Tangirala, campbell
- Siddarth, Guha, (Missouri City, TX)
- Shivani, Guha, (Missouri City, TX)
- Ravikiran, Komirisetty, (Irvine, CA)
- Simoni, Maniar, (Grapevine, TX)
- preetham, bachina, (Pleasanton, CA)
- Anish, Natarajan, (Newbury Park, CA)
- Deepankar, Gupta, (Naperville, IL)
- Shraman, Sen, (Morrisville, NC)
- Bharati, Dalal, NC
- Aman, Dalal, NC
- Pratik, Reddy, (Somerset, NJ)
- Nishant, Chittari, (New Albany, OH)
- Janani, Sridhar, (Cupertino, CA)

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- Nadarajan,Janakan, (Troy, MI)
- Shashank,Mahesh, (Nagpur, India)
- Varun,Singh, (Tampa, FL)
- Indumathi,Prakash, (Sharon, MA)
- Dhruva,mambapoor, (Austin, TX)
- Milan,Kodali, (Plano, TX)
- rohan,rereddy,garland
- Meghal,Gupta, (Cupertino, Ca)
- Vamsi,Subraveti, (Nashville, TN)
- Shruthi,Santhanam, (Suwanee, GA)
- Sanjna,Khanna, (Piscataway, NJ)
- Pranav,Kotilingam, (North Brunswick, NJ)
- Himanvi,Kopuri, (Denver, CO)
- Shreya,Bellur, (Dunlap, IL)
- Anish,Bose, (Cedar Park, TX)
- Shriitha,Gunturu, (Aurora, CO)
- Tanishq,Kancharla, (Middlebury, CT)
- Bhavana,Muppavarapu, (Buffalo Grove, IL)
- Nick ,Patel, (NYC, NY)
- Shreyaa,Raghavan, (Sharon, MA)
- Keerti ,Vajrala, (Aurora, CO)
- Dhivya ,Senthil Murugan,CO
- Rahul,Madala, (Chantilly, VA)
- Anish,Madala, (Chantilly, VA)
- Lakshmi,Kothuri, (Naperville, IL)
- Uma,Kothuri, (Naperville, IL)
- Bharat,Kabra, (Edison, NJ)
- Shraeya,Madhu, (Clarksburg, MD)
- Vishal ,Purohitham, (Sanford, FL)
- Mridula,Gopal, (SAT, TX)
- praneeth ,prathi, (Shrewsbury, MA)
- Mounisha,Kovour, (Algonquin, IL)
- Balaji,Venkatesan, (Weymouth, MA)
- Sneha,Prasath, (Westford, MA)
- Ambika,Goel, (Woburn, MA)
- Nisha,Goel, (Woburn, MA)
- Sriraj,Atluri, (Weston, FL)
- Sreedhar,Atluri, (Weston, FL)
- Vishik,Bhalla, (Nashua, NH)
- Vishal,Gullapalli, (Wayne, NJ)
- Anna,Nixon, (Portland, OR)
- Suresh,Aiyer, (Herndon, VA)
- Ashwath,Raj, (San Diego, CA)
- Shaheel,Mitra, (Cincinnati, OH)
- YASHWANTH NALLA (CONCORD, NC)
- Sahana,Aiyer (Herndon, VA)
- Neha,Dudipala (Lewis Center, OH)
- Rishita,Saladi (Campbell, CA)
- Sreenu,Pamidi (Shrewsbury, MA)
- Arya,Goutam (Burlingame, CA)
- Desigamoorthy,Nainar (Champaign, IL)
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- Kala,Mani (Lisle, IL)
- Akshay ,Venkat (Leawood, KS)
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- Ranjan veludandi (Spring, TX)

Thanks to all who attempted to solve the problem of the month. The Math Column team is looking forward to your continued interest and increased participation.