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Math Trick

Mental Calculation: Multiplication by 11

The Trick

Mentally calculate:

$$\begin{array}{lll}
 32 \times 11 = & 63 \times 11 = & 28 \times 11 = \\
 59 \times 11 = & 41 \times 11 = & 75 \times 11 =
 \end{array}$$

These are two digit numbers multiplied by 11.

Let \overline{ab} be a two-digit number in the general form where a and b are digits.

If we follow the given steps below, we will be able to calculate $\overline{ab} \times 11$ faster.

Example 1

Calculate 25×11 .

Step 1: Separate a and b : $a \square b$. Here \square represents a digit. In this example, we have $2 \square 5$.

Step 2: Place $a + b$ in the box between a and b .

In this example, place $2 + 5 = 7$ between 2 and 5: $2 \square 7 5$.

We are done: $25 \times 11 = 275$.

If $a + b > 9$, there is a carry of 1. Then place the unit digit of $a + b$ in the box between a and b , and add 1 to a .

Example 2

Calculate 86×11 .

Step 1: Separate 8 and 6: $8 \square 6$.

Step 2: Place the unit digit 4 of $8 + 6 = 14$ between 8 and 6, and add 1 to 8 yielding $9 \square 4 6$.

Then $86 \times 11 = 946$.

Example 3

Calculate 97×11 .

Step 1: Separate 9 and 7: $9 \square 7$.

Step 2: Place the unit digit 6 of $9 + 7 = 16$ between 9 and 7, and add 1 to 9 yielding $10 \square 6 7$.

We obtain $97 \times 11 = 1067$.

The trick works for a number with three or more digits.

Example 4

Calculate 345×11 .

Step 1: Separate 3 and 5: $3 \square \square 5$. There are two digits between 3 and 5.

Step 2: Place $4 + 5 = 9$ in the right box, and place $3 + 4 = 7$ in the left box: $3 \square 9 \square 5$.

We are done: $345 \times 11 = 3795$.

Example 5

Calculate 789×11 .

Step 1: Separate 7 and 9: $7 \square \square 9$.

Step 2: $8 + 9 = 17$. Place 7 in the right box. $7 + 8 = 15$ and $15 + 1 = 16$ where 1 is carried from 17. Place 6 in the left box. Add 1, carried from 16, to 7 yielding 8. Finally we obtain $8 \square 6 \square 7 9$.

So $789 \times 11 = 8679$.

Example 6

Calculate 1234×11 .

Step 1: Separate 1 and 4: $1 \square \square \square 4$.

Step 2: Place $3+4=7$ in the rightmost box, place $2+3=5$ in the box in the middle, and place $1+2=3$ in the leftmost box: $1 \boxed{3} \boxed{5} \boxed{7} 4$.

We obtain $1234 \times 11 = 13574$.

For a number having three or more digits, we should mentally fill the boxes from the left to the right.

Why Does This Work?

The following two figures show why this works.

$$\begin{array}{r} a\ b \\ \times 1\ 1 \\ \hline a\ b \\ a\ b \\ \hline a\ \uparrow\ b \\ a+b \end{array} \qquad \begin{array}{r} a\ b\ c \\ \times 1\ 1 \\ \hline a\ b\ c \\ a\ b\ c \\ \hline a\ \uparrow\ \uparrow\ c \\ a+b\ \uparrow\ b+c \end{array}$$

Practice Problems I

- $12 \times 11 =$ $78 \times 11 =$ $44 \times 11 =$
- $39 \times 11 =$ $99 \times 11 =$ $46 \times 11 =$
- $82 \times 11 =$ $38 \times 11 =$ $96 \times 11 =$
- $52 \times 11 =$ $67 \times 11 =$ $11 \times 11 =$

Practice Problems II

- $123 \times 11 =$ $468 \times 11 =$ $579 \times 11 =$
- $987 \times 11 =$ $654 \times 11 =$ $321 \times 11 =$
- $2345 \times 11 =$ $6789 \times 11 =$ $8642 \times 11 =$

Math Competition Skill

Monkeys Eat Bananas

Introduction Problems

The following are three problems quite popular in junior math competitions.

We assume that all monkeys eat bananas at the same rate.

Problem 1

Three monkeys eat 3 bananas in 3 minutes. How many bananas do 9 monkeys eat in 9 minutes?

Problem 2

Three monkeys eat 3 bananas in 3 minutes. How many bananas do 4 monkeys eat in 5 minutes?

Problem 3

a monkeys eat c bananas in b minutes. How many bananas do d monkeys eat in e minutes?

Problem 1 and Its Variations

In the first problem all numbers are multiples of 3. It is relatively easy to solve. Three steps are shown to obtain the answer.

Solution to Problem 1:

Step 1:

List the numbers of monkeys, minutes, and bananas in a table.

Step 2:

Change the number of monkeys to 9 keeping 3 minutes unchanged. Proportionally change the number of bananas.

Step 3:

Change the number of minutes to 9 keeping 9 monkeys unchanged. Proportionally change the number of bananas.

	Monkeys	Minutes	Bananas
<i>Step 1</i>	3	3	3
<i>Step 2</i>	9	3	9
<i>Step 3</i>	9	9	27

The answer is 27. That is, 9 monkeys eat 27 bananas in 9 minutes.

First Variation of the Problem

In the first variation, the number of minutes is to be determined. The problem may be:

Three monkeys eat 3 bananas in 3 minutes. In how many minutes do 9 monkeys eat 81 bananas?

The similar three steps are presented in the solution.

Solution:

Step 1:

List the numbers of monkeys, minutes, and bananas.

Step 2:

Change the number of monkeys to 9 keeping 3 minutes. Proportionally change the number of bananas.

Step 3:

Change the number of bananas to 81 keeping 9 monkeys. Proportionally change the number of minutes.

	Monkeys	Minutes	Bananas
<i>Step 1</i>	3	3	3
<i>Step 2</i>	9	3	9
<i>Step 3</i>	9	27	81

The answer is 27. That is, 9 monkeys eat 81 bananas in 27 minutes.

Second Variation of the Problem

In the second variation, the number of monkeys is to be determined. The problem may be:

Three monkeys eat 3 bananas in 3 minutes. How many monkeys eat 243 bananas in 81 minutes?

Solution:

Step 1:

List the numbers of monkeys, minutes, and bananas.

Step 2:

Change the number of minutes to 81 keeping 3 monkeys. Proportionally change the number of bananas.

Step 3:

Change the number of bananas to 243 keeping 81 minutes. Proportionally change the number of monkeys.

	Monkeys	Minutes	Bananas
<i>Step 1</i>	3	3	3
<i>Step 2</i>	3	81	81
<i>Step 3</i>	9	81	243

The answer is 9. That is, 9 monkeys eat 243 bananas in 81 minutes.

Problem 2 and Its Variations

In the second problem not all numbers are multiples of 3. We have five steps now.

Solution to Problem 2:

Step 1:

List the numbers of monkeys, minutes, and bananas.

Step 2:

Change the number of monkeys to 1 keeping 3 minutes. Proportionally change the number of bananas.

Step 3:

Change the number of minutes to 1 keeping 1 monkey. Proportionally change the number of bananas.

Step 4:

Change the number of monkeys to 4 keeping 1 minute. Proportionally change the number of bananas.

Step 5:

Change the number of minutes to 5 keeping 4 monkeys. Proportionally change the number of bananas.

	Monkeys	Minutes	Bananas
<i>Step 1</i>	3	3	3
<i>Step 2</i>	1	3	1
<i>Step 3</i>	1	1	$\frac{1}{3}$
<i>Step 4</i>	4	1	$\frac{4}{3}$
<i>Step 5</i>	4	5	$\frac{20}{3}$

The answer is $\frac{20}{3}$. That is, 4 monkeys eat $\frac{20}{3}$ bananas in 5 minutes.

First Variation of the Problem

In the first variation we ask for the number of minutes. The problem may be:

Three monkeys eat 3 bananas in 3 minutes. In how many minutes do 5 monkeys eat 10 bananas?

Solution:

Step 1:

List the numbers of monkeys, minutes, and bananas.

Step 2:

Change the number of monkeys to 1 keeping 3 minutes. Proportionally change the number of bananas.

Step 3:

Change the number of minutes to 1 keeping 1 monkey. Proportionally change the number of bananas.

Step 4:

Change the number of monkeys to 5 keeping 1 minute. Proportionally change the number of bananas.

Step 5:

Change the number of bananas to 10 keeping 5 monkeys. Proportionally change the number of minutes.

	Monkeys	Minutes	Bananas
<i>Step 1</i>	3	3	3
<i>Step 2</i>	1	3	1
<i>Step 3</i>	1	1	$\frac{1}{3}$
<i>Step 4</i>	5	1	$\frac{5}{3}$
<i>Step 5</i>	5	6	10

The answer is 6. That is, 5 monkeys eat 10 bananas in 6 minutes.

In the last step we have to figure x out such that $\frac{5}{3} \cdot x = 10$. It is obvious that $x = 6$.

Second Variation of the Problem

In the second variation we ask for the number of monkeys. The problem may be:

Three monkeys eat 3 bananas in 3 minutes. How many monkeys eat 20 bananas in 15 minutes?

Solution:

Step 1:

List the numbers of monkeys, minutes, and bananas.

Step 2:

Change the number of monkeys to 1 keeping 3 minutes. Proportionally change the number of bananas.

Step 3:

Change the number of minutes to 1 keeping 1 monkey. Proportionally change the number of bananas.

Step 4:

Change the number of minutes to 15 keeping 1 monkey. Proportionally change the number of bananas.

Step 5:

Change the number of bananas to 20 keeping 15 minutes. Proportionally change the number of monkeys.

	Monkeys	Minutes	Bananas
Step 1	3	3	3
Step 2	1	3	1
Step 3	1	1	$\frac{1}{3}$
Step 4	1	15	5
Step 5	4	15	20

The answer is 4. That is, 4 monkeys eat 20 bananas in 15 minutes.

Problem 3 and Its Variations

In the third problem there are no concrete numbers. Instead there are variables represented by lower-case letters. Five steps are shown in the solution.

Solution to Problem 3:

Step 1:

List the numbers of monkeys, minutes, and bananas.

Step 2:

Change the number of monkeys to 1 keeping b minutes. Proportionally change the number of bananas.

Step 3:

Change the number of minutes to 1 keeping 1 monkey. Proportionally change the number of bananas.

Step 4:

Change the number of monkeys to d keeping 1 minute. Proportionally change the number of bananas.

Step 5:

Change the number of minutes to e keeping d monkeys. Proportionally change the number of bananas.

	Monkeys	Minutes	Bananas
Step 1	a	b	c
Step 2	1	b	$\frac{c}{a}$
Step 3	1	1	$\frac{c}{ab}$
Step 4	d	1	$\frac{cd}{ab}$
Step 5	d	e	$\frac{cde}{ab}$

The answer is $\frac{cde}{ab}$. That is, d monkeys eat $\frac{cde}{ab}$ bananas in e minutes.

Similarly we have two variations: 1. asking for the number of minutes; 2. asking for the number of monkeys.

First Variation of the Problem

a monkeys eat c bananas in b minutes. In how many minutes do d monkeys eat e bananas?

Solution:

Step 1:

List the numbers of monkeys, minutes, and bananas.

Step 2:

Change the number of monkeys to 1 keeping b minutes. Proportionally change the number of bananas.

Step 3:

Change the number of minutes to 1 keeping 1 monkey. Proportionally change the number of bananas.

Step 4:

Change the number of monkeys to d keeping 1 minute. Proportionally change the number of bananas.

Step 5:

Change the number of bananas to e keeping d monkeys. Proportionally change the number of minutes.

	Monkeys	Minutes	Bananas
Step 1	a	b	c
Step 2	1	b	$\frac{c}{a}$
Step 3	1	1	$\frac{c}{ab}$
Step 4	d	1	$\frac{cd}{ab}$
Step 5	d	$\frac{abe}{cd}$	e

The answer is $\frac{abe}{cd}$. That is, d monkeys eat e bananas in

$\frac{abe}{cd}$ minutes.

In the last step we have to figure x out such that $\frac{cd}{ab} \cdot x = e$. Obviously, $x = \frac{abe}{cd}$.

Second Variation of the Problem

a monkeys eat c bananas in b minutes. How many monkeys eat e bananas in d minutes?

Solution:

Step 1:

List the numbers of monkeys, minutes, and bananas.

Step 2:

Change the number of monkeys to 1 keeping b minutes. Proportionally change the number of bananas.

Step 3:

Change the number of minutes to 1 keeping 1 monkey. Proportionally change the number of bananas.

Step 4:

Change the number of minutes to d keeping 1 monkey. Proportionally change the number of bananas.

Step 5:

Change the number of bananas to e keeping d minutes. Proportionally change the number of monkeys.

	Monkeys	Minutes	Bananas
Step 1	a	b	c
Step 2	1	b	$\frac{c}{a}$
Step 3	1	1	$\frac{c}{ab}$
Step 4	1	d	$\frac{cd}{ab}$
Step 5	$\frac{abe}{cd}$	d	e

The answer is $\frac{abe}{cd}$. That is, $\frac{abe}{cd}$ monkeys eat e bananas in d minutes.

Practice Problems I

- Four monkeys eat 4 bananas in 4 minutes. How many bananas do 6 monkeys eat in 6 minutes?
- Four monkeys eat 4 bananas in 4 minutes. In how many minutes do 10 monkeys eat 12 bananas?
- Four monkeys eat 4 bananas in 4 minutes. How many monkeys eat 10 bananas in 5 minutes?
- x monkeys eat z bananas in y minutes. How many bananas do u monkeys eat in v minutes?
- x monkeys eat z bananas in y minutes. In how many minutes do u monkeys eat v bananas?
- x monkeys eat z bananas in y minutes. How many monkeys eat u bananas in v minutes?

Practice Problems II

In the following problems we assume that all people work at the same rate.

- Two men paint a house in 3 days. How many houses can be painted by 3 men in 4 days?
- Two men paint a house in 3 days. In how many days do 6 men paint 5 houses?
- Two men paint a house in 3 days. How many men are needed to paint 14 houses in 12 days?
- If x workers can complete a job in y days, how many days will it take one worker to complete z jobs?
- If a workers can complete b jobs in one day, how many workers are needed to complete 1 job in c days?
- If one worker can complete u jobs in v days, how many days are needed to complete v jobs by u workers?

A Problem from a Real Math Competition

Today's problem comes from British Columbia Senior High School Mathematics Contest (BCSHSMC).

Problem

(BCSHSMC 2000 Preliminary Round Problem 13)

If it takes x builders y days to build z houses, how many days would it take q builders to build r houses? Assume that these builders work at the same rate as the others.

Answer: $\frac{xyr}{zq}$

Solution:

We are equipped with the method to solve this problem. We readily have the answer with these five steps:

	Builders	Days	Houses
Step 1	x	y	z
Step 2	1	y	$\frac{z}{x}$
Step 3	1	1	$\frac{z}{xy}$
Step 4	q	1	$\frac{zq}{xy}$
Step 5	q	$\frac{xyr}{zq}$	r

Practice Problem

(MathCounts 2008 National Sprint Problem 19)

If m workers can complete a job in d days, how many days will it take n workers, working at the same rate, to complete one-third of the job? Express your answer as a common fraction in terms of d, m and n .

Answers to All Practice Problems in Last Issue

Math Trick: Mental Calculation

Practice Problems I

625	9025	4225
3025	225	7225
1225	5625	2025

Practice Problems II

11025	38025	570025
60025	308025	156025
801025	416025	1010025

Divisibility by 2, 4, 8, etc.

Practice Problems I

- The numbers divisible by 4 are:
1992 2000 188 24680 1768 2004

2. The numbers divisible by 8 are:
5000 2008 24680 87456 24016
3. The numbers divisible by 16 are:
90000 69136 62480 23456 12624

Practice Problems II

1. 1212122112
2. 896

A Problem from a Real Math Competition

The answer is 300 meters. The detailed solution is given. Let m and t be the uphill speeds of Maria and Tony, respectively.

When Maria and Tony meet, Maria has run $700 + 70$ meters, and Tony has run $700 - 70$ meters. However, Maria has run 70 meters at the downhill speed which is double the uphill speed.

We count 70 meters as $70 \div 2 = 35$ meters as if she had run at her uphill speed during the downhill period. Note that the distance ratio is equal to the speed ratio. So we have $\frac{m}{t} = \frac{735}{630}$. That is, $\frac{m}{t} = \frac{7}{6}$.

When Maria reaches the bottom, she has run a distance equivalent to $700 + 700 \div 2 = 1050$ meters as if he had run all the way at her uphill speed. At this time, Tony has run a distance equivalent to $1050 \times \frac{6}{7} = 900$ meters as if

he had run all the way at his uphill speed. However, he has run 700 meters at his uphill speed, and the rest ($900 - 700 = 200$ meters) at his downhill speed. He has actually run $200 \times 2 = 400$ meters during his downhill period.

Therefore, Tony is $700 - 400 = 300$ meters behind Maria.

Solutions to Creative Thinking Problems 16 to 18

16. $2 + 3 \neq 5$

When you do the addition incorrectly, $2 + 3$ is not equal to 5.

What is your answer?

17. Another 81 Balls

Note that $81 = 3^4$. Divide 81 balls into 3 groups with each group of 27 balls. Weigh one group against the second group.

If the scale is in balance, the bad ball is in the third group. If the scale is not in balance, the bad ball is in the group which is lighter. We can always identify a group of 27 balls, in which the bad ball is.

Then divide this group of 27 balls into three groups with each of 9 balls. With the second weighing, we can

similarly identify a group of 9 balls, in which the bad ball is.

With the third weighing, we can identify a group of 3 balls, in which the bad ball is.

With the fourth weighing, we can identify the bad ball.

I have to weigh four times to identify the bad ball.

18. A Popular Pattern

Count how many 1s, how many 2s, etc. in each number.

Count for the first number 1: One 1. We have 11 as the second number.

Count for the second number 11: Two 1s. We have 21 as the third number.

Count for the third number 21: One 1, One 2. Then we have 1112 as the fourth number.

Count for the fourth number 1112: Three 1s, One 2. We have 3112 as the fifth number.

Count for 3112: Two 1s, One 2, One 3. We have 211213 as the sixth number.

Counting for 212223 we have 114213 as the answer.

The number after 114213 is 31121314.

Creative Thinking Problems 19 to 21

19. Strange Symbols

This is a wonderfully designed pattern even though the symbols look strange. What comes next?



20. A Number Reversed

Find \overline{ABCDE} , a five-digit number such that

$$\begin{array}{r} A B C D E \\ \times \quad \quad \quad 4 \\ \hline E D C B A \end{array}$$

where different letters represent different digits.

21. Four Boys Crossing a River

Four boys, namely Al, Bob, Carl, and Dan, are going to cross a river in a dark night. The narrow bridge can carry only two boys at once. They have to use a flashlight because of the darkness. Unfortunately they have only one flashlight. When two boys have crossed the bridge one boy must carry back the flashlight for other boys to use. It is known that Al needs 5 min for crossing the bridge. Bob needs 10 min, Carl needs 20 min, and Dan needs 25 min. Two boys together need the time of the slower one. What is the minimum time for all the four boys to cross the river?

(Solutions will be presented in the next issue.)