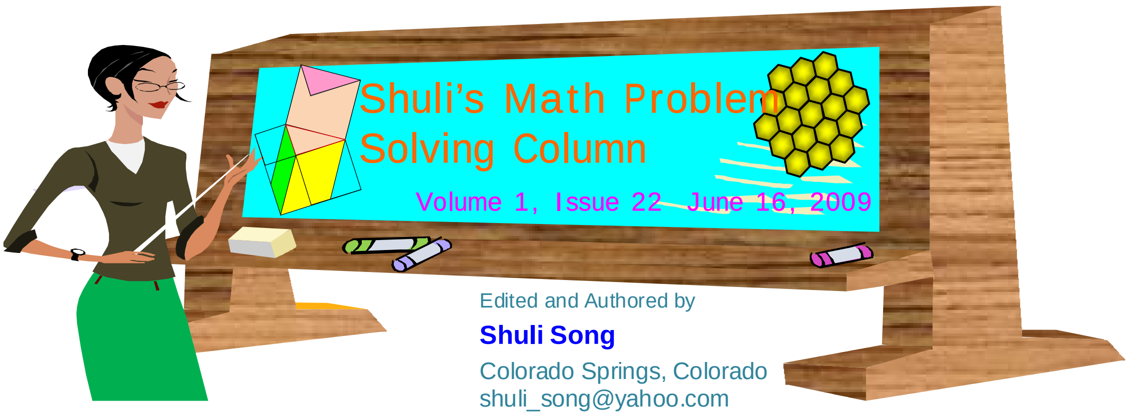

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Contents

Math Trick

Mental Calculation: × 9, 99, 999, etc.

The Trick

You may know how to do the following multiplications:

$75\times 9 =$ $456\times 99 =$ $864\times 999 =$

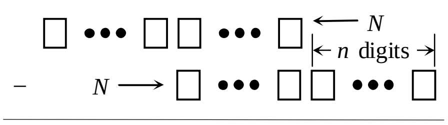
$1234\times 9999 =$ $13579\times 99999 =$

There is a short cut to multiply a number by a number with several 9's.

Let N and n be two positive integers.

The multiplications can be presented in the general form $N\times \overbrace{99\cdots 9}^{n\ 9\text{'s}}$.

We may obtain the answer by subtracting N from N shifting n positions to the left:



The following shows the shortcut through examples.

Example 1

Calculate 36×9.

Subtract:

	3	6	
−		3	6
	3	2	4

So $36\times 9 = 324$.

Example 2

Calculate 58×99.

Subtract:

	5	8		
−			5	8
	5	7	4	2

We obtain $58\times 99 = 5742$.

Example 3

Calculate 1234×999.

Subtract:

	1	2	3	4			
−				1	2	3	4
	1	2	3	2	7	6	6

We have $1234\times 999 = 1232766$.

The next example is from Canadian Mathematics Competition (CMC).

Example 4

(CMC 1998 Pascal (Grade 9) Problem 23)

The numbers 123456789 and 999999999 are multiplied. How many of the digits in the final result are 9's?

Answer: 0

Solution:

$$123456789\times 999999999$$
$$= 123456789000000000 - 123456789$$
$$= 123456788876543211.$$

There is no digit 9 in the product.

Why Does This Work?

The following expression shows why this works.

$$N \times \overbrace{99\cdots9}^{n\ 9\text{'s}} = N \times (\overbrace{100\cdots0}^{n\ 0\text{'s}} - 1)$$

$$= N \times \overbrace{100\cdots0}^{n\ 0\text{'s}} - N = N\underbrace{\overline{00\cdots0}}_{n\ 0\text{'s}} - N$$

Practice Problems I

$321 \times 9 =$	$654 \times 99 =$	$987 \times 999 =$
$9 \times 83 =$	$99 \times 93 =$	$999 \times 8642 =$
$99 \times 374 =$	$999 \times 47 =$	$258 \times 9999 =$
$123456 \times 9999 =$	$97531 \times 999 =$	$2468 \times 99 =$
$999^2 =$	$99^2 =$	$9999^2 =$

Practice Problems II

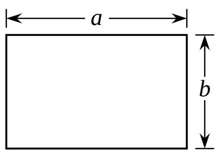
1. If 987654321 and 999999999 are multiplied, how many of the digits in the final result are 0's?
2. If 12345679 and 99999999 are multiplied, how many of the digits in the final result are 9's?
3. If 12345679 and 999999999 are multiplied, how many of the digits in the final result are 9's?

Math Competition Skill

Perimeter of a Polygon with Perpendicular Sides

Perimeters of Rectangles and Squares

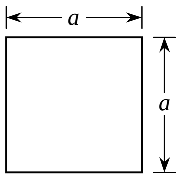
In the rectangle shown, two different sides are a and b in length.



The perimeter of the rectangle is

$$a + b + a + b = 2 \cdot (a + b).$$

If $a = b$, the rectangle becomes a square of side length a.

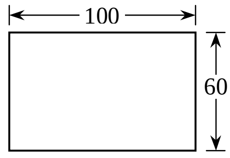


The perimeter of the square is

$$a + a + a + a = 4a .$$

Example 1

What is the perimeter in meters if a rectangular field is 100 meters long and 60 meters wide?



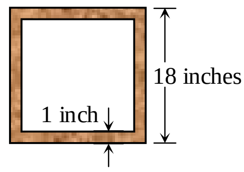
Answer: 320

Solution:

$$2 \cdot (100 + 60) = 320 .$$

Example 2

A picture frame is a square shown below. The length of one side is 18 inches. The border in all four sides is 1 inch wide. What is the perimeter of the inside square in inches?



Answer: 64

Solution:

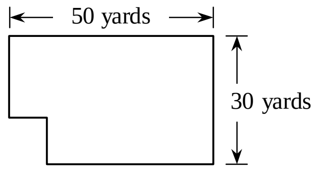
The side length of the inside square is $18 - 2 \times 1 = 16$. The perimeter is $4 \times 16 = 64$.

Perimeters of Polygons with Perpendicular Sides

In this section any two neighboring sides of a polygon are perpendicular to each other.

Example 3

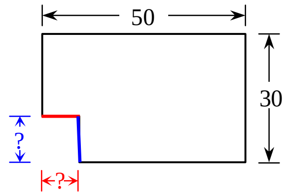
A swimming pool has the shape as shown. What is the perimeter of the pool in yards?



Answer: 160

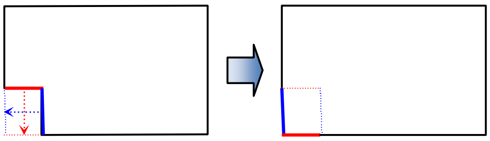
Solution:

It looks that we cannot calculate the perimeter because we don't know the lengths of the two segments colored below in red and blue respectively.



Do we have to know them?

Look at the figure below. I move the two colored segments as shown:



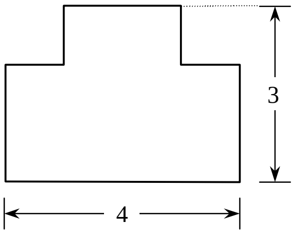
I have changed the shape of the pool. Have I changed the perimeter?

Obviously, I haven't.

So the perimeter of the pool is the same as the rectangle with side lengths of 50 yards and 30 yards. Therefore, the answer is $2\cdot(50+30)=160$ yards.

Example 4

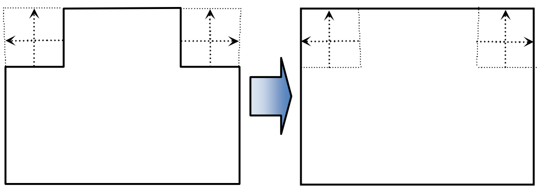
Find the perimeter of the shape shown.



Answer: 14

Solution:

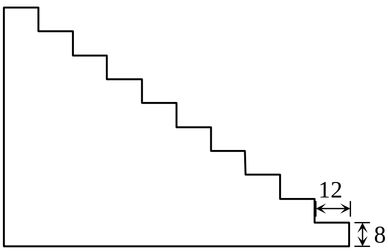
We also move some segments without changing the perimeter.



The shape has the perimeter same as the rectangle with side lengths of 3 and 4. So the answer is $2\cdot(4+3)=14$.

Example 5

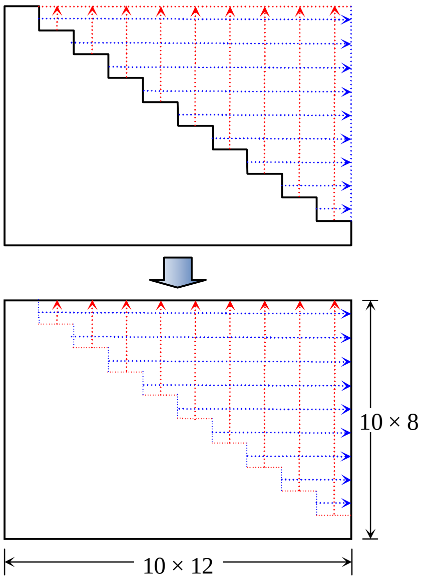
The figure shows the cross-section of 10 stairs. If one stair is 8 inches high and 12 inches wide, what is the perimeter of the shape in inches?



Answer: 400

Solution:

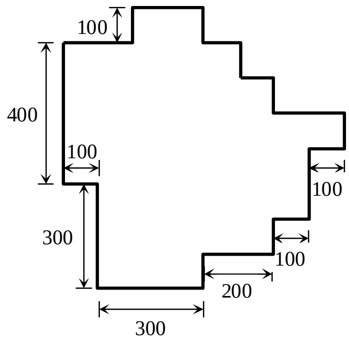
As shown in the figure below, the perimeter of the original shape is the same as the rectangle which is $10\times 12=120$ inches long and $10\times 8=80$ inches wide.



Therefore, the answer is $2\cdot(120+80)=400$.

Example 6

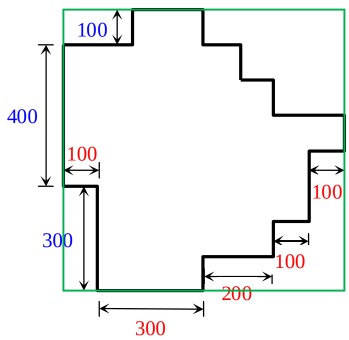
A park has a shape as shown. The dimensions are given in meters. In the morning Dr. Math runs around the park three times. How long does Dr. Math run in a week in miles if he runs five days a week.



Answer: 30

Solution:

The perimeter of the park is the same as the rectangle drawn below in green.



We have to find the side lengths of the green rectangle.

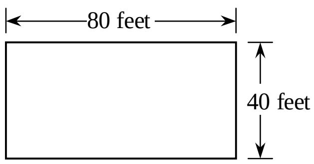
We can find the length of the rectangle, which is the sum of the three blue numbers: $100+400+300=800$, and the width of it, which is the sum of the five red numbers: $100+300+200+100+100=800$.

Then the perimeter of the park is $2\times(800+800)=3200$ meters.

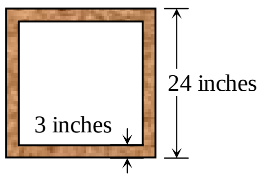
Note that 1 mile is 1600 meters. So the perimeter of the park is 2 miles. Dr. Math runs $3\times 2=6$ miles in one morning. For 5 days, he runs $5\times 6=30$ miles.

Practice Problems I

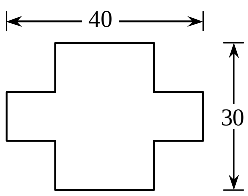
1. What is the perimeter in feet if a rectangular field is 80 feet long and 40 feet wide?



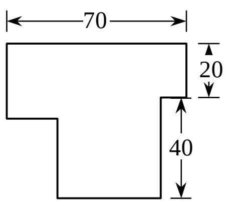
2. A picture frame is a square shown below. The length of one side is 24 inches. The border in all four sides is 3 inches wide. What is the perimeter of the inside square in inches?



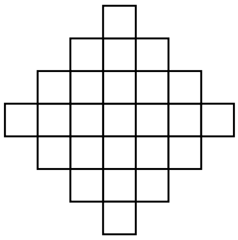
3. What is the perimeter of the polygon shown?



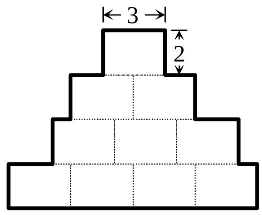
4. What is the perimeter of the shape shown?



5. If each smallest square has side length 1, what is the perimeter of the shape?



6. The following shape consists of ten congruent rectangles with side lengths of 2 and 3. What is the perimeter of the shape?



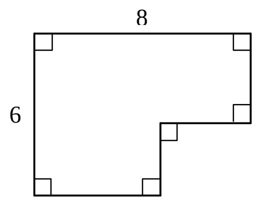
Practice Problems II

1. (AMC8 1986 Problem 13)

The perimeter of the polygon shown is

A) 14 B) 20 C) 28 D) 48

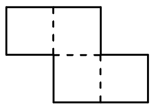
E) cannot be determined from the information given



2 (AMC8 1990 Problem 15)

The area of this figure is 100 cm². Its perimeter in cm is

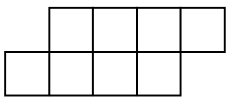
A) 20 B) 25 C) 30 D) 40 E) 50



[Figure consists of four identical squares]

3. (AMC8 1992 Problem 22)

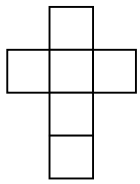
Eight 1 by 1 square tiles are arranged as shown so their outside edges form a polygon with a perimeter of 14 units. Two additional tiles of the same size are added to the figure so that at least one side of each tile is shared with a side of one of the squares in the original figure. Which of the following could be the perimeter of the new figure?



A) 15 B) 17 C) 18 D) 19 E) 20

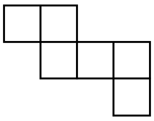
4. (MathCounts 2000 Chapter Countdown Problem 5)

A hollow cube has a volume of 0.008 cm^3. What is the number of meters in the perimeter of the figure formed by opening and flattening the cube as shown? Express your answer as a decimal to the nearest tenth.



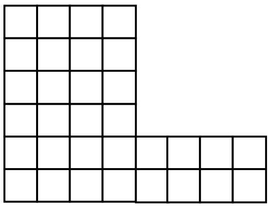
5. (MathCounts 2007 State Countdown Problem 54)

The area of this region formed by six congruent squares is 294 square centimeters. What is the perimeter of the region, in centimeters?



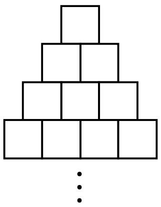
6. (MathCounts 2004 National Sprint Problem 21)

This L-shape piece of land is made of 32 unit squares. By constructing a fence, only along drawn segments in the figure, the one large piece of land can be divided into two congruent smaller pieces of land, each with area 16 square units. What is the number of units in the perimeter of one these smaller pieces of land?



7. (MathCounts 1999 National Sprint Problem 30)

Squares with sides 1 centimeter long are arranged as shown, each row containing one more square than the row above it. How many centimeters are in the perimeter of the figure formed by arranging 210 squares in this fashion?



A Problem from a Real Math Competition

Today's problem comes from American Regions Mathematics League (ARML).

(ARML 1992 Problem T-7)

Compute $\sqrt{(111{,}111{,}111{,}111)(1{,}000{,}000{,}000{,}005)+1}$.

Answer: 333,333,333,334

Solution:

There are twelve 1's in the first number, and there are eleven 0's in the second number.

Let us study from small numbers.

There is one 1 in the first number, and there is no 0 in the second number: $\sqrt{(1)(15)+1}=\sqrt{16}=4$.

There are two 1's in the first number, and there is one 0 in the second: $\sqrt{(11)(105)+1}=\sqrt{1156}=34$.

There are three 1's in the first number, and there are two 0's in the second: $\sqrt{(111)(1005)+1}=\sqrt{111556}=334$.

We have found the pattern. Then the answer to the problem is 333,333,333,334.

Practice Problem

Prove that 111,111,111,111,222,222,222,222 is the product of two consecutive integers.

Answers to All Practice Problems in Last Issue

Mental Calculation

437	432	476
832	864	1036
1394	1710	1376
2544	3618	2352
3213	4964	4736

Divisibility by 11

1. The numbers divisible by 11 are:
   121 213532 1331 123321 456456 2750
2. 4 5 3 9 3 4 0 0 4 0
3. Yes 4. 1
5. 6 6. $a=b=7$ 7. 9
8. The alternate digit difference is 0.

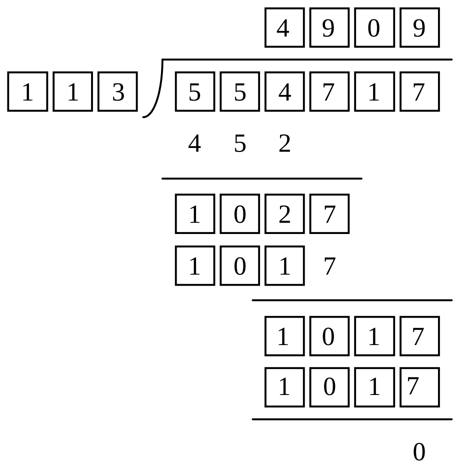
A Problem from a Real Math Competition

1. $4\sqrt{30}$ 2. A 3. C 4. C

Solutions to Creative Thinking Problems 58 to 60

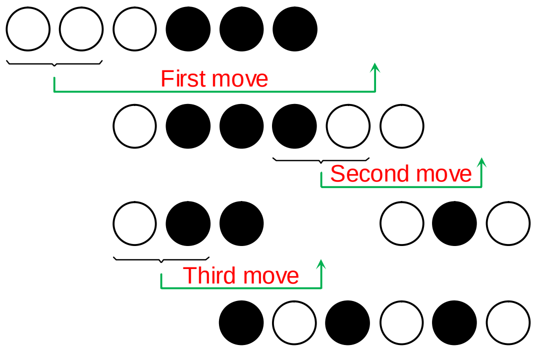
58. A Division

$452=113\times 2^2$. The divisor must be 113, 226, 0r 452. Noting the 7 given we know that the divisor must be 113.



59. Moving Checkers

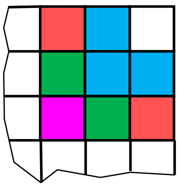
This is the solution:



60. A Checkerboard Game

Let us study from the right-top corner.

If the checker is at one of the three blue squares, the next player will win immediately. We call these blue squares *winning positions*.

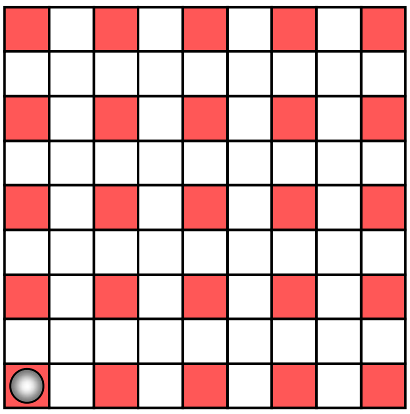


Look at one of the two red squares. If the checker is there, the next player will have to move it to a winning position, which is the blue square sharing a side with it. So we call the red squares *losing positions* accordingly.

Look at one of the two green squares. If the checker is there, the next player can move it to a losing position, which is the red square sharing a side with it. We also call the green squares winning positions.

Now look at the pink square. If the checker is there, the next player will have to move it to a winning position – either one of the two green squares or the blue square diagonal to it. The pink square is a losing position.

Continuing this analysis we can identify all losing positions colored with red in the figure below. All other squares are winning positions.



Since the checker is at a losing position initially, the first player will lose if the second player follows the strategy. The first player has always to move the checker to a wining position (white square). The second player is always able to move it to a losing position (red square).

Clues to Creative Thinking Problems 61 to 63

61. Four Trominos to One

You don't need a clue for this problem, do you?

62. Magic Circles

Find the sum of the numbers in the other 10 circles?

63. Two Smart Students of Dr. Math

If the sum is 2, Al will know $m = n = 1$. If the sum is 3, Al will know $m = 1$ and $n = 2$. If the sum is 4, Al cannot know m and n because $1 + 3 = 4$ and $2 + 2 = 4$. Even though Al doesn't know m and n in this case, he cannot be sure Bob doesn't know m and n either. If Dr. Math might have picked $m = 1$ and $n = 3$, the product would be 3. Then Bob would know $m = 1$ and $n = 3$.

Continue on the analysis.

Creative Thinking Problems 64 to 66

64. One Million Words on One Square Inch

How do you write down a million words on a piece of paper of only one square inch?

65. Greatest 11-Digit Number

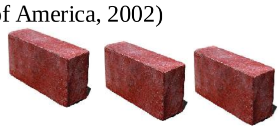
Delete 20 digits in the following number

1234567891011121314151617181920

such that the remaining digits form the possible greatest 11-digit number keeping the order.

66. Diagonal of a Brick

Give you three identical rectangular bricks and a long ruler. How do you measure the diagonal length of the bricks directly. You are not supposed to measure the three sides, and then calculate the diagonal length with the Pythagorean Theorem. (The excellent puzzle is from "*The Inquisitive Problem Solver*, Paul Vaderlind, Richard Guy, and Loren Larson, The Mathematical Association of America, 2002)



(Clues and solutions will be given in the next issues.)