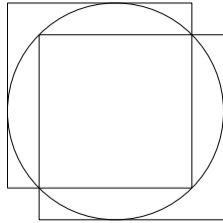


AMC/AIME Prep Club Pre-Test

August 2022

1. Calvin was asked to evaluate $37 + 31 \times a$ for some number a . Unfortunately, his paper was tilted 45 degrees, so he mistook multiplication for addition (and vice versa) and evaluated $37 \times 31 + a$ instead. Fortunately, Calvin still arrived at the correct answer while still following the order of operations. For what value of a could this have happened?
2. How many 4-digit numbers have only odd digits?
3. The side length of a cube is increased by 100%. What is the increase in the volume of the cube, as a percentage?
4. A certain two-digit number is equal to twice the sum of its digits. What is the product of its digits?
5. In her last game, Mary bowled 199, raising her average from 177 to 178. To raise her average to 179, what must she bowl in her next game?
6. The surface of a rectangular $9 \times 10 \times 11$ block is painted red, and the block is then cut into cubes with side length 1. Find the number of cubes that have exactly one red face.
7. Find the number of positive integers $1 \leq n \leq 1000$ such that $n \cdot n$ is a perfect square.
8. A drawer has 5 pairs of socks. Three socks are chosen at random. What is the probability that there is a pair among the three?
9. In $\triangle ABC$, $AB = 16$, $AC = 15$, and $BC = 13$. Points D and E lie on AB and AC so that DE bisects the area and the perimeter of $\triangle ABC$. Compute DE .
10. What is the only ordered pair of real numbers (x, y) that satisfies $7^x - 11y = 0$ and $11^x - 7y = 0$?
11. Let $ABCD$ be an isosceles trapezoid with $AD = BC$, $AB = 6$, and $CD = 10$. Suppose the distance from A to the centroid of $\triangle BCD$ is 8. Compute the area of $ABCD$.
12. Let $S = \{1, 2, 3, 4, 5, 6\}$, and consider all two-element subsets of S . What is the maximum number of these subsets we can choose so that no three, say X, Y, Z , satisfy $X \cup Y \cup Z = S$?
13. If p, q , and r are primes with $pqr = 7(p + q + r)$, find $p + q + r$.

14. Let c be the smallest real solution to the equation $3^x = x + 2$. To six decimal places, $c = -1.87213$. Calculate the value of 3^{3^c} , rounded to the nearest hundredth.
15. Two unit squares cover a circle of radius r , as shown below. The corresponding sides of the squares are parallel. What is r ?



16. For positive integer m, n , let $\gcd(m, n)$ denote the largest positive integer that is a factor of both m and n . Compute $\gcd(1, 91) + \gcd(2, 91) + \gcd(3, 91) + \cdots + \gcd(91, 91)$.